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Short history of stock markets and stochastic finances

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Abstract. This text will be published as a part of the Introduction to the book *Financial mathematics*, 1st Edition, ISTE PRESS, Elsevier, 2016.

Financial markets are sometimes identified with stock markets; sometimes they are differentiated with the view that on the financial markets we can trade only securities and on the stock market we can trade other values, such as real estate, property and currency. The stock market is also called stock exchange. A stock exchange is a market for different kinds of securities, including stocks, bonds, shares as well as payment documents. As for the randomness, the situation is such that the prices in the financial market, more specifically, on the stock exchange, are affected by many external factors that cannot be predicted in advance and cannot be controlled completely. This is mainly a consequence of economic circumstances, for example, of the state of the world economy and of the local economy, production levels in some sectors, and the balance between supply and demand. It may be the weather and climate factors affecting, for example, a certain type of agricultural products, or it may be the activities of large exchange speculators. Since stock prices at any given time are random, over time they accordingly become random processes. Of course, the same situation occurred even in those days when exchange existed, but the theory of random processes has not yet been established. Recall that the Chicago Stock Exchange began operating 21 March 1882.

As for the theory of random processes, curiously enough, its founder was not a mathematician but botanist Robert Brown, who in 1827 discovered under a microscope the process of chaotic motion of flower pollen in water. The nature of this phenomenon remained unclear for long time, and only in the late 19th—early 20th Century it was realized that it is one of the manifestations of the thermal motion of atoms and molecules, and to explore this phenomenon we need methods of probability theory. Appropriate random process was eventually called the Brownian motion, and then Wiener process, according to the name of the famous mathematician Norbert Wiener who not only constructed integral with respect to this process but also wrote hundreds of articles on probability theory and mathematical statistics, Fourier series and integrals, potential theory, number theory and generalized harmonic analysis. He is also called the "father of cybernetics" for his book "Cybernetics: or Control and Communication in the Animal and the Machine" [1], first published in 1948.

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He helped to develop a system of air defense of the USA. Note that the initial framework for the analysis of randomness in the change in stock prices was established by French mathematician and economist Louis Bachelier, who in 1900 in his doctoral thesis "Théorie de la spéculation" [2] made the attempt to describe the stock price by means of stochastic process $S = S_t, t \geq 0$ with the increments $\Delta S_t = S_{t+\Delta t} - S_t$ of order $\sqrt{\Delta t}$, in some probabilistic sense. Such a process is a prototype of the Wiener process but the Bachelier's model had a crucial disadvantage: the prices in this model could be negative. In fact, Bachelier's model can be described as $S_t = S_0 + \mu t + \sigma W_t$, where W is a Wiener process. Nevertheless, Bachelier's discovery of the "effect of Δt" in fluctuations of the value of shares under a large number of economic factors and due to the central limit theorem became later the key point in the construction of the general theory of random diffusion processes. Although for some time Bachelier's works had been forgotten, after many years they were rightly remembered and highly appreciated, and now the main representative congresses on financial mathematics are named World Congresses of the Bachelier Finance Society. At the beginning, the mathematical study of the Brownian motion (Wiener process) was produced in the papers of physicists, namely Albert Einstein and Marian Smoluchowski, and then it was widely studied by mathematicians, including Norbert Wiener.

A very interesting person in economic theory and, to some extent, in financial mathematics is Russian mathematician Leonid Kantorovich, a specialist in functional analysis. In 1938, he provided advice to plywood plant how to use their machines in the most effective way to minimize the waste of plywood. Over time, Kantorovich realized that such very particular problem can be generalized to the problem of maximization of the linear form depending on many variables and containing a large number of restrictions that have the form of linear equalities and inequalities. He also realized that the enormous number of economic issues can be reduced to the solution of such problems. In 1939, Kantorovich published the paper "Mathematical Methods of Organizing and Planning Production" [3], describing the problems of the economy that can be solved by his method and thus laid the foundation of mathematical programming. His contribution directly into financial mathematics is that he found such an interesting coincidence: the best prices, including the prices of financial assets, are at the same time the prices supplying market equilibrium. Then his conclusions were obtained independently by US economists, and in 1975 he received the Nobel Prize in economics together with Tjalling C. Koopmans "for their contribution to the theory of optimal allocation of resources".

Financial mathematics has received a new impetus for development in 1965, when at the initiative of mathematician and economist Leonard Savage, who "rediscovered" Bachelier's work, American economist Paul Samuelson, who would also subsequently become the winner of the Nobel Prize in economics, has suggested to describe share prices with the help of geometric Brownian motion $S_t = S_0 e^{\mu t} e^{\sigma W_t - \sigma^2 t/2}$, whose advantage is to be non-negative and even strictly positive with probability 1 [4]. Over time, the model of geometric Brownian motion was substantially generalized. In particular, we can consider jump-diffusion process or Levy process, that is homogeneous process with independent increments, or semimartingale, instead of the Wiener process.

Finally, in 1968, there was a significant economic and financial event: the prices of gold and other precious metals were "released". The history of this issue is as follows: from 1933 to 1976, the official price of gold was under control of the Department of the Treasury of the United States federal government. Now it is managed, in a certain sense, by the London Stock Exchange. In 1944, the price of gold was at the level of 35 USD per troy ounce (31.1034768 g) and from time to time increased or decreased under the influence of the devaluation of the dollar, world crises or wars. The price of gold increased due to the increasing of the demand for gold as a raw material for production of electronics and radio-technics, the jewelry industry, medicine and other purposes. But often the price of gold grew as the result of speculative transactions on the stock exchange and as the result of the creation of the highly liquid assets by central banks of different countries. In 1961, Western Europe countries created a "golden pool", which included central banks of the UK, Germany, France, Italy, Belgium, The Netherlands, Switzerland and the Bank of New York. This pool was created in order to stabilize the world prices for gold, but in 1968, after the devaluation of the British pound, UK spent 3,000 tons of gold to regulate interior prices for gold, and after this the gold pool collapsed. From that time, the price of gold is determined by the market, i.e., by the demand and the supply.

Free gold prices led to additional random components in the financial markets, and the stochastic finance theory started to develop very intensively both as a theoretical science and as a tool for the daily management of banking and stock exchange activities. An additional factor that contributed to its development was the opening of the first stock exchange in 1973 on which option contracts were traded. In the same year, two works that led to the revolution in financial calculations of option prices were published. It was the paper of Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities" [5], and the paper of Robert Merton, "Theory of Rational Option Pricing" [6]. In October 1997, R. Merton (Harvard University) and M. Scholes (Stanford University) were awarded the Nobel Prize in economics. (F. Black died in 1995, and the Nobel Foundation awards prizes only to living scientists). Briefly, the Black-Scholes formula evaluates "fair" option price. The Black-Scholes-Merton model is very useful in making investment decisions, but principally does not guarantee profit without risk. Conceptually, the Black-Scholes formula can be explained as follows: the option price equals the expected future asset price minus the expected cash price, or as the difference of two binary options: an asset-or-nothing call minus a cash-or-nothing call. The concept of fair price is based on the concept of arbitrage-free market. We should pay attention to the point that the real market can be modeled in various ways, and its properties will be different in different models. For example, the same market can be modeled as complete and incomplete, but the only way to determine which model suits the best is to verify them in practice. Typically, the construction of several models of the market and the consideration of several trading strategies are expensive problems, and the art of a financial analyst consists, in particular, of choosing the correct model. Note also that the models constructed for financial mathematics are not situated aside all other science and practice. Indeed, they are used in biology, weather forecasting, climatology and the study of changes in the mobile electrical circuits communication because the processes in these fields very often have the same features.

The description of modern financial models is based both on the theory of random processes and stochastic analysis (theory of martingales, stochastic integration, Itô formula, Girsanov's theorem, theory of stochastic differential equations, martingale representations and elements of Malliavin calculus) and on basic facts of functional analysis (topological, Banach and Hilbert spaces, linear functionals, Hahn-Banach theorem etc.).

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