

CONTROL OF MULTISCALE SYSTEMS WITH CONSTRAINTS
 4. CONTROL OF THE EVOLUTION OF NUCLEAR SYSTEMS
 ON THE BASIS OF THE PRINCIPLE OF DYNAMICAL
 HARMONIZATION

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Abstract. We consider the application of the principle of dynamical harmonization to the description and the realization of the electromagnetic control of the processes of self-organization of the internal structure of nuclear systems and isomers. We present the main experimental results confirming the following theoretical conclusions: the synthesis of a wide spectrum of elements, the positive energy yield of nuclear installations, and the control of the half-life of unstable nuclear systems can be realized with the help of the optimization of mass (entropic) forces used in the installations.

Contents

1	Introduction	36
2	Collective states of fractal nuclear isomers and the dynamics of nuclear clusters	40
2.1	Collective models of fractal nuclear isomers and clusters	40
2.2	Deformation of nuclear isomers under the action of electromagnetic and entropic fields and the half-lives of unstable nuclei	45
2.3	Specific features of the evolution of the structure of a nucleus and the equations of dynamical harmonization	46
3	Beta-processes at the evolution of a structure of nuclei under the action of mass forces	54
3.1	Probability of beta-processes for fractal isomers	54
3.2	Regularities of the decays of nuclear systems with regard for their coherence	56
3.3	Evolution of the activities in a mixture of fractal isomers	57

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4	The model based on a system of iterated functions (SIF) for the dynamics of a system of many nuclei under the action of mass forces	60
4.1	Statistical properties of a system of nuclei and the model based on SIF	61
5	Realization of the processes of collective synthesis and the control of nuclear processes	66
5.1	Control of unstable nuclear systems	70
5.1.1	Statistical properties of experimental series of the decay activities for isotopes	70
5.1.2	Comparison of theoretical and experimental data on the activity of a mixture of isomers	72
5.2	On a promising technology of nuclear fusion in coherent multi-scale systems	77
6	Conclusion	88
	Appendix. Algorithms of processing of experimental time series	89
	Informational distances and definition of nonsteadiness of time series	89
	Regularizing properties of fractional derivatives	90
	Classify the attractors	92
	References	94

1 Introduction

This work continues the cycle “Control of multiscale systems with constraints” and presents some approaches to the control of the evolution of nuclear systems on the basis of the general principle of self-organization of systems with varying constraints, namely the principle of dynamical harmonization.

The clarification of laws of the evolution of complicated systems, which would allow one to reliably forecast it and, on the whole, to control the evolution, is one of the oldest and most important problems attracting a steady attention of researchers. In the cycle of works [1–3], the general approach to their solution on the basis of the fundamental notions of constraints, dynamical harmonization controlling the evolution, and geometry is advanced.

The principle of dynamical harmonization [1–2] asserts that the self-organization of systems with variable constraints between elements, being under the action of mass forces that cause the coherent accelerations exceeding the dissipative chaotic accelerations considerably, is always directed so that the structure of the system in the course of its collective evolution will transit to a state with the most free dynamics. In work [3], the above formulation of the principle was given in the geometric interpretation: the system of particles in the course of its evolution curves, as a result of the superthreshold acceleration, the space-time, where it is placed; in its turn, the space-time becoming curved indicates the direction of free motion to particles and the direction of evolution for the whole system of particles. In other words, the evolution of the system

occurs so that the world lines in the curved space-time will correspond to the trajectories of a proper free motion of particles of the system.

The dynamical harmonization implies directly that the input system of independent particles in the inertial reference system, being coherently accelerated, becomes a system of particles interacting by means of a collective field.

We should note that the scope of the principle of dynamical harmonization may be expanded because the processes of evolution of matter are closely related to the properties of the physical vacuum. Coherent acceleration of the system defines restructuring of space-time, that is, the physical vacuum, against which the substance is evolving, restructuring, in turn, the physical vacuum. The principle of dynamical harmonization governs not only the self-organization processes of matter, but also the processes of self-organization of a vacuum-matter system. Therefore, we can formulate a generalized inverse Hertz principle stating that not only system connections form the curvature of space-time, but also the change of space-time curvature in the area, characterized by the scale of the coherence of a system, reorganizes the connections ensemble in the system. Thus, self-organization of the matter occurs with the help of a feedback through the physical vacuum.

In order to initiate the process of self-organization, it is necessary to create a mass force that will ensure the optimal coherent acceleration, the noninertialness of the reference system, and the transition of the system of particles in a nonequilibrium state due to the nonzero flows in the phase space. It was shown in [1–3] that the coherent acceleration and the noninertialness of a reference system are the reason for the evolution of the internal structure of the system of particles with the total mass number A and for the appearance of some coherent share with a mass number A_{cog} in the system, i.e., the reason for the appearance of the nonzero order parameter $\eta = A_{cog}/A$.

The practically unique available means for the formation of mass forces and for the control of the processes of evolution in systems of particles on the macroscopic scale are electromagnetic and entropic fields. Namely they are the main tool of a controlling technology.

In addition, the specific feature of such technologies consists in that the energy source inducing a change of the structure of a system is not the energy of external drivers, but the internal mass defect energy of the system itself. The low energy of external fields must be used only for the control of and the initiation of the processes of self-organization with a desired direction.

The problems of the controlled synthesis of new structures of various nature and the optimization of drivers for the realization of such processes are of importance from the applied and theoretical viewpoints. For example, the most promising directions in the creation of energy sources able to ensure the permanently increasing demand of the humanity for the energy are related to the development of optimal drivers for the synthesis of nuclear structures.

Till the recent time, the approaches to the problem of controlled nuclear fusion were based on the ideas of that a change of nuclear structures requires one to have the initiating and controlling sources (drivers), whose bulk energy densities at their outputs are comparable with nuclear ones (MeV/nucleon) by scale or somewhat higher ones (due to the high characteristic values of binding energy). It is clear that the physical technologies satisfying such requirements are not controlling technologies, strictly speaking, at any realization.

The general analysis of the processes of evolution of complicated systems was performed in [1–3] and indicates definitely the possibility to initiate the processes of synthesis, which can be realistically called the technologies of control of a wide circle of objects spreading from nuclear to biological ones.

It is natural that the models of nuclear systems involving their internal structure and constraints occupy the central place in the efficient solution of problems of the synthesis of new nuclear structures.

According to our ideas [2], the nuclei with mass number A and charge number Z (especially heavy ones) can form a wide spectrum of nuclear clusters including the fragments of a “web” consisting of the nuclear matter with standard density (bound nucleons, alpha-particles, or other stable nuclear formations with mass number A_{str}) and the amorphous part formed by the ordinary “nuclear fluid”. In addition to masses and charge numbers, the nuclear clusters are characterized also by the value of fractal dimension D_f .

The order parameter $\eta = A_{cog}/A$ becomes the essential characteristic of a nuclear system. It gives the ratio between the masses of the coherent and amorphous parts of a nuclear cluster and is connected with the fractal dimension of nuclear structures D_f by the relation [2]

$$\eta = \frac{3 - D_f}{D_f - 1} \quad \text{or} \quad D_f = \frac{\eta + 3}{\eta + 1}. \quad (2.1)$$

In the generalized drop model of a nucleus, the binding energy in the Bethe–Weizsäcker form with modified coefficients is as follows (see [1–2]):

$$B(A, Z, D_f) = \left(c_0 - c_3 \left(1 - \frac{2Z}{A} \right)^2 \right) A - c_1 A^{2/3} - c_2 \frac{Z^2}{A^{1/3}} + \frac{c_p}{A^{1/2}} \begin{cases} 1, & Z = 2l, N = 2k \\ 0, & A = 2k + 1 \\ -1, & Z = 2l + 1, N = 2k + 1 \end{cases}, \quad (2.2)$$

$$c_0 \approx \left(58.4 - 42.6 \left(\frac{A}{A_{str}} \right)^{-2} \left(\frac{1}{D_f} - \frac{1}{3} \right) \right), \quad c_1 \approx 18.56 \left(\frac{A}{A_{str}} \right)^2 \left(\frac{1}{D_f} - \frac{1}{3} \right),$$

$$c_2 \approx 0.71 \left(\frac{A}{A_{str}} \right)^{-1} \left(\frac{1}{D_f} - \frac{1}{3} \right), \quad c_3 \approx 23.7 \left(\frac{A}{A_{str}} \right)^{-2} \left(\frac{1}{D_f} - \frac{1}{3} \right).$$

The order parameter determines the shares of nucleons entering the amorphous part of the nucleus with a lower density and the binding energy of the order of $B((1 - \eta)A, (1 - \eta)Z, D_f)$ and the coherent part of “web threads,” which are constructed of the elementary stable structures with mass number A_{str} and charge number Z_{str} , and the binding energy of the order of $B\left(\eta A, \frac{Z_{str}}{A_{str}} \eta A, D_f\right)$.

It is clear that the dependence of the binding energy of nuclei on their internal structure (on the fractal dimension of nuclei, the order parameter, and other parameters sensitive to a structure) leads to the possibility for the

stability of the fractal isomers of nuclei in processes of evolution of nuclear systems initiated and controlled by the coherent acceleration to vary in wide limits.

In the present work, we analyze, theoretically and experimentally, the possibilities to develop the technologies of control of the synthesis of nuclear structures and their properties. As a tool in the creation of a coherent acceleration in a system of particles, we can use the vector potential and the appropriate pulses of the strength of an electromagnetic field, which act on charged particles and are practically homogeneous on nuclear spatial scales. Such pulses are well described by a pulse of the vector potential $\vec{A}(t)$, which is independent of coordinates and determines the strength of the electric component of the electromagnetic field $\vec{E} = -\frac{\partial \vec{A}(t)}{\partial t}$, as well as the variation of the momenta of all charged particles by a value proportional to $\vec{A}(t)$. The action of homogeneous pulses of the electromagnetic field $\vec{A}(t)$ on the system of particles is a mass force and realizes a coherent impact on all particles (see [3]). By exceeding some threshold, this action ensures the coherent acceleration of particles and initiates the evolution of constraints in a system of particles [1–3, 4].

It is clear that the control of systems can be most simply realized near the threshold of their stability. Therefore, the first studies of the controlled evolution of a nuclear matter are performed here by the example of the controlled change of a structure of unstable nuclei with the purpose to neutralize their radioactivity.

The theme of the present work is the investigation of the spectrum of nuclear fractal isomers arising and evolving as a result of the action of electromagnetic pulses on a system of unstable radioactive nuclei.

Here, we refine the dependence of the binding energy of nuclear fractal clusters on their deformation and study, theoretically and experimentally, the possibility to control the processes of synthesis with the help of pulses of an electromagnetic field acting on a system of beta- and alpha-active nuclei. Theoretically and experimentally, we study the laws of decay of unstable nuclear systems, i.e., the laws of the dynamics of natural random processes and the possibilities to control them.

Traditionally, the law of variation of the radioactivity in the course of time is assumed to be exponential and practically independent of external conditions in wide limits, since each elementary act of decay of any of the large collection of radioactive nuclei is considered random and independent.

Such ideas are natural and proper in the inertial closed systems of particles. But the creation of the coherent acceleration [3] induces the curving of the space-time and the appearance of a collective interaction even in a system of initially independent particles. The breaking of the independence of random events of decay of nuclear isomers results in that, as will be demonstrated in what follows, the exponential law of decay of unstable clusters approaches a power dependence, as the coherent acceleration and the mass force increase. The degree of deviation of the decay law from the exponential one allows one to estimate both the curvature of the space-time that arose due to the action of electromagnetic pulses on a macroscopic system with radioactive admixtures and the efficiency of the control of a nuclear system. The performed theoretical and experimental studies can underlie the development of new technologies of neutralization of the radioactivity and the synthesis of new nuclear structures.

2 Collective states of fractal nuclear isomers and the dynamics of nuclear clusters

In the above-presented relations for the binding energy, nuclei and clusters are assumed spherical. We now analyze the influence of the internal fractal structure of nuclei on their collective properties, deformation, and energy levels.

2.1 Collective models of fractal nuclear isomers and clusters

The internal fractal structure of fractal nuclear formations leads to the dependence of the density distribution in a nucleus and the nucleus boundary erosion on the order parameter. Moreover, they are described by the laws with power asymptotics, rather than by a law similar to the Fermi distribution function. In order to estimate the density distribution $\rho_{nuc}(r)$ and the generalized Woods–Saxon potential $V_\eta(r)$ in nuclear fractal isomers, it is natural to replace the exponential functions by their coherent generalizations [5] $\exp_\eta(x) = (1 + \eta x)^{1/\eta}$ related to a reduction of the dynamical degrees of freedom in the system and the development of a structure with some order parameter:

$$\begin{aligned}\rho_{nuc}(r) &= \frac{\rho_0}{1 + \exp_\eta\left(\frac{1}{1-\eta} \frac{r - R_\eta(A)}{\Delta}\right)}, \\ V_\eta(r) &= \frac{V_0}{1 + \exp_\eta\left(\frac{1}{1-\eta} \frac{r - R_\eta(A)}{\Delta}\right)}, \\ R_\eta(A) &= R_{str} \left(\frac{A}{A_{str}}\right)^{\frac{\eta+1}{3+\eta}},\end{aligned}\tag{2.3}$$

where $V_0 \approx -50\text{MeV}$ is the potential depth, $R_\eta(A)$ is the radius of a nuclear cluster, and Δ is the parameter characterizing an erosion of the nucleus edge, $0 \leq \eta < 1$ is the parameter of coherence of the system ($\eta = 0$ corresponds to the absence of a coherence). The ordinary exponential function and the Fermi function arise in the limiting case of the infinite numbers of particles and degrees of freedom. The function $\exp_\eta(x)$ tends to the exponential function $\exp(x)$ as $\eta \rightarrow 0$. In Fig. 1, we show the distribution function for various values of a coherence degree.

To refine the shapes of potentials of a nucleus, it is necessary to find a change of the binding energy at a deformation of a nucleus. In the simplest case, the deformation of a nucleus transfers it from a sphere to an ellipsoid of rotation [6] with the deformation of the nucleus $\delta = \frac{a-b}{a+b}$, where a and b are the lengths of the major and minor semi-axes of the ellipsoid. The ellipsoid volume V_{ab} is given by the simple formula $V_{ab} = \frac{4\pi}{3}ab^2$ in terms of the semi-axes. For the incompressible nuclear fluid, we can write

$$a = r_0 A^{1/D_f} (1 + \varepsilon), \quad b = r_0 A^{1/D_f} \frac{1}{\sqrt{1 + \varepsilon}} \approx r_0 A^{1/D_f} \left(1 - \frac{1}{2}\varepsilon\right),$$

$$\delta = \frac{(1 + \varepsilon)^{3/2} - 1}{(1 + \varepsilon)^{3/2} + 1} \approx \frac{3}{4}\varepsilon. \quad (2.4)$$

The parameters of a deformation of the nucleus enter directly only the surface energy and the Coulomb interaction energy.

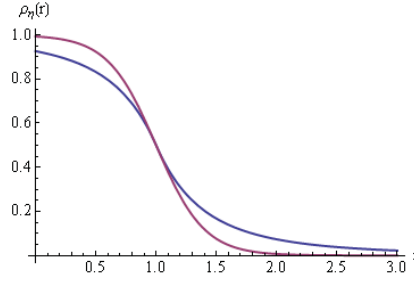


Fig. 1. Density distribution in a nucleus for the coherent state of a fractal cluster and for a continuous drop without structure.

The ellipsoid surface area is a function of its semiaxes, and the surface energy of a nucleus of the ellipsoidal shape (it is proportional to the ellipsoid surface area) is as follows:

$$\begin{aligned} W_S &= \sigma S_{ab} \approx c_1 (A, D_f, \delta) A^{2/3}, \\ c_1 &= 18.56A^2 \left(\frac{1}{D_f} - \frac{1}{3} \right) \left(1 + \frac{a}{b} \frac{\arcsin \left(\sqrt{1 - \left(\frac{b}{a} \right)^2} \right)}{\sqrt{1 - \left(\frac{b}{a} \right)^2}} \right) \\ &\approx 18.56A^2 \left(\frac{1}{D_f} - \frac{1}{3} \right) \left(1 + \frac{2}{5}\varepsilon^2 \dots \right) \end{aligned} \quad (2.5)$$

It is easy to calculate also the Coulomb energy of a charged ellipsoid:

$$\begin{aligned} W_q &\approx c_2 \frac{Z(Z-1)}{A^{1/3}}, \\ c_2 &= 0.71A^{-\left(\frac{1}{D_f} - \frac{1}{3} \right)} \frac{\ln \left(\frac{a + \sqrt{a^2 - b^2}}{b} \right)}{\sqrt{a^2 - b^2}} \\ &\approx 0.71A^{-\left(\frac{1}{D_f} - \frac{1}{3} \right)} \left(1 - \frac{1}{5}\varepsilon^2 \right). \end{aligned} \quad (2.6)$$

Formula (2) with coefficients accounting for the dependence on a deformation determines the dependence of the specific binding energy on the mass numbers, charge numbers, fractal dimension, and deformation.

$$\begin{aligned}
B(A, Z, D_f, \delta) &= B(A, Z, D_f) + B_\delta(A, Z, D_f), \\
B_\delta(A, Z, D_f) &= 18.56A^{2\left(\frac{1}{D_f} - \frac{1}{3}\right)} \left(\frac{2}{5}\varepsilon^2 + \dots\right) A^{2/3} \\
&\quad - 0.71A^{-\left(\frac{1}{D_f} - \frac{1}{3}\right)} \left(\frac{1}{5}\varepsilon^2 + \dots\right) \frac{Z^2}{A^{1/3}}, \quad (2.7)
\end{aligned}$$

The sign of the additive $B_\delta(A, Z, D_f)$ to the binding energy due to a deformation determines, as is known, the stability of a nucleus. If the creation of a deformation requires some energy, then the stability is present, and the internal surface and Coulomb forces return the nucleus in the initial stable state. If the deformation of a nucleus causes a decrease of the energy of the system in a new state, then the nucleus is not stable, and the transition into a deformed state can occur spontaneously with a release of some free energy in the system. In this case, the stability boundary is determined by the condition $B_\delta(A, Z, D_f) = 0$ and depends, as usual, on the fission parameter of a nucleus ξ .

However, the parameter ξ is now determined not only by the charge and the mass number of a nucleus, but also by its structure. So, we have $\xi = \frac{Z^2}{A^{3/D_f}}$, and the stability boundary is determined by the relation (the value of right-hand side can vary in the interval from 49 to 52, by depending on the chosen approximation of the binding energy in the model of liquid drop):

$$\frac{Z^2}{A^{3/D_f}} = 49. \quad (2.8)$$

If the internal structure of a nucleus is absent, then $D_f = 3$, and we arrive at the well-known result. The appearance of an internal structure with $D_f < 3$ leads to the extension of the stability domain of nuclei. To simplify the analysis of the stability boundary, we use the stability line

$$Z_{st}(A, D_f) = A \left(2 + 0.015A^{2/3} \left(\frac{A}{A_{str}} \right) \left(\frac{1}{D_f} - \frac{1}{3} \right) \right)^{-1}$$

and obtain the stability curve on the plane (D_f, A) :

$$A^{2-\frac{3}{D_f}} = 49 / \left(2 + 0.015A^{2/3} \left(\frac{A}{A_{str}} \right) \left(\frac{1}{D_f} - \frac{1}{3} \right) \right)^2. \quad (2.9)$$

The stability curve is shown in Fig. 2 as the intersection of the plane $x = 1$ with the surface $x = \xi/49 = x(A, D_f)$.

For the liquid drops without internal structure ($D_f = 3$), the stability boundary is close to the mass number of an uranium nucleus, whereas, for the nuclei with internal structure, there exists the value of fractal dimension, below which all nuclei are stable.

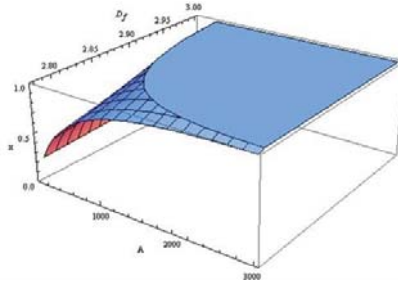


Fig. 2. Curve of stability to decays and the fission parameter surface for fractal nuclei.
A decrease in the dimension causes the extension of the stability domain.

The typical map of levels of the specific binding energy for deformed fractal isomers is presented in Fig. 3. The oscillatory properties of nuclei and their deformation are key in the description of the nuclear processes and, first of all, the processes of decay and fusion of nuclei. The high-frequency oscillations of the surface of a nucleus play the role of a noise acting on the low-frequency oscillations of the surface. The latter can be represented as the deformational oscillations of a nucleus (e.g., such as the oscillation of nucleus' parts relative to one another in the process of their formation).

The main contribution to a deformation of the nucleus with charge number Z and mass number A is given by the dynamics that can be presented by the interaction of its two approximately equal parts that possess the parameters A_1, Z_1 and $A_2 = A - A_1, Z_2 = Z - Z_1$ and are placed near two centers.

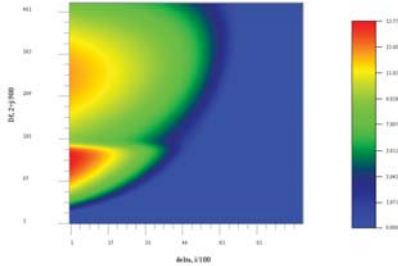


Fig. 3. Map of levels of the specific binding energy of fractal deformed isomers of a nucleus (^{90}Sr) on the deformation (δ)-fractal dimension (D_f) plane.

The interaction of these components can be considered as the motion of some effective particle in the interaction potential from the mentioned parts. The coordinate of the nuclear process r is the distance between the centers of masses of the parts, into which the nucleus is effectively separated, or is its deformation. The potential, in which the effective particle moves, is determined by the parameters of the nucleus and its binding energy. The adequacy of such description is based on the assumption that the time needed to attain the thermal equilibrium in the system of nucleonic degrees of freedom is essentially less than the characteristic time scales of the collective motion related to the process of deformation of the nucleus.

The process of evolution of the system along the coordinate of reaction or deformation δ occurs in the potential field $U_{eff}(\delta)$, which has the main

minimum at some point, near which the oscillations are realized. The ground state is separated from the other states by the reaction barrier with height W_f (see [6]). With regard for (3) and (1), we have

$$W_f = \frac{98}{135}(1-x)^3 \sigma_{surf} A^{2/D_f}, \quad x = \xi/49, \quad \xi = \frac{Z^2}{A^{3/D_f}}. \quad (2.10)$$

The dependence of the barrier height on the fractal dimension and the mass number (2.10) is given in Fig. 4, and the typical shapes of the potential are shown in Fig. 5.

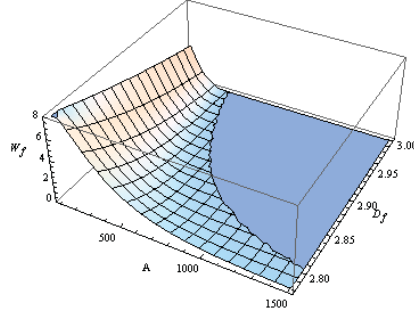


Fig. 4. Reaction barrier height in MeV as a function of the mass number and the fractal dimension of a nucleus.

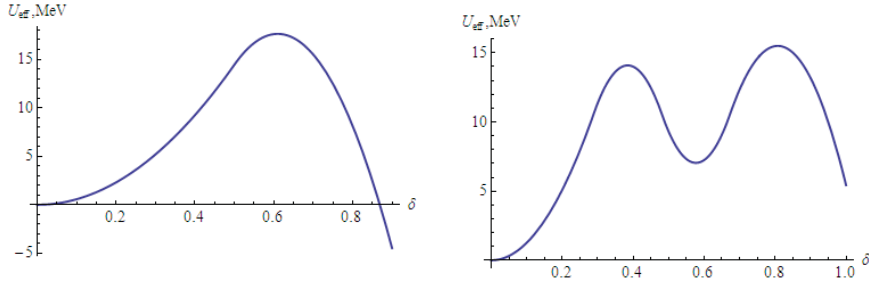


Fig. 5. Potentials $U_{def}(\delta)$ for the effective particle describing the large-scale evolution of a nucleus.

The distance between the centers in the unstable equilibrium at the barrier top

$$r_m = 1.15 A_1^{1/D_f} \left(1 + \sqrt{\frac{5}{4\pi}} \delta_1 \right) + 1.15 A_2^{1/D_f} \left(1 + \sqrt{\frac{5}{4\pi}} \delta_2 \right), \quad (2.11)$$

where δ_1 and δ_2 are the small parameters of deformation of the appropriate parts of the nucleus. Approximately, we can take $r_m \approx \frac{2}{3}(R_1 + R_2)$, i.e., $\delta \approx 0.68$. If the reaction coordinate $r \approx r_f \approx R_1 + R_2$ ($\delta \leq 0.9$) after the maximum at the barrier top, the potential decreases almost down to the initial state. The further increase in the coordinate causes the decay.

In the right part of Fig. 5, we show the case where the additional minimum appears on the potential curve. This minimum is observed in many nuclei (e.g., for ^{198}Hg and plutonium ^{240}Pu) and corresponds to the isomeric state of a nucleus.

2.2 Deformation of nuclear isomers under the action of electromagnetic and entropic fields and the half-lives of unstable nuclei

If a nucleus is split as a result of its instability, then the free energy is released in the form of the kinetic energy of outgoing fission fragments. In a simple model of the decay of a nucleus, including the alpha - decay, its probability $\lambda = \frac{\ln 2}{\tau_{1/2}}$ is equal to the product of the collision frequency ν of a fragment of the nucleus, which can leave the nucleus with potential barrier W_A (boundary of the nucleus) and the probability of the passage of a possible fragment through the potential barrier D_W (transparency of the barrier)

$$\frac{\ln 2}{\tau_{1/2}} = \nu D_W \quad \text{or} \quad \tau_{1/2} = \frac{\ln 2}{\nu D_W}. \quad (2.12)$$

The collision frequency ν of a fragment can be estimated in terms of the nucleus radius R_A and its velocity u in the nucleus:

$$\nu = \frac{u}{2R_A} \approx \frac{u}{2r_0 A^{1/D_f}} \approx \frac{c}{2r_0 A^{1/D_f}} \left(\frac{2(W_{kA} + W_A)}{\mu_A c^2} \right)^{1/2}, \quad (2.13)$$

where W_{kA} and μ_A are, respectively, the kinetic energy and the reduced mass of a fragment (e.g., alpha - particle). Substituting the characteristic values of $W_A \approx 35$ MeV and the kinetic energy ≈ 5 MeV in relation (13), we obtain the collision frequency $\nu \approx 10^{21} \text{ s}^{-1}$ for nuclei with $A \approx 200$.

The quantum-mechanical solution of the problem of the passage of particles through a potential barrier implies that the passage probability (coefficient of transparency of the barrier)

$$D_W = \exp \left(-2 \sqrt{\frac{2\mu_A}{\hbar^2}} \int_{R_A}^{r_e} (W(r) - W_{kA})^{1/2} dr \right). \quad (2.14)$$

In the approximation of a purely Coulomb barrier ($B_q = \frac{Z_1 Z_2 e^2}{r_0 A^{1/D_f}}$ is the energy of a Coulomb barrier) with regard for the delocalization of the wave function, the barrier passage probability can be presented in the form

$$D_W \approx \exp_q \left(-2 \sqrt{\frac{2\mu_A}{W_{kA}} \frac{Z_1 Z_2 e^2}{\hbar^2} \left(\frac{\pi}{2} - 2 \frac{W_{kA}}{B_q} \right)} \right),$$

$$q \approx \sqrt{1 + \frac{P_S}{P_{dis}}} \approx \sqrt{1 + \alpha \left(\frac{a_{cog}}{a_{dis}} \right)^2}. \quad (2.15)$$

In this relation, we consider a variation of the half-life due to an increase of the energy of fragments caused by structural changes of the nucleus and the delocalization of wave functions, which is reflected by the nonequilibrium parameter q . We have the following approximate formula for the evaluation of the half-life:

$$\lg(\tau_{1/2}) \approx \frac{9.54 Z_1^{0.6}}{\sqrt{W_{kA}, \text{MeV}}} - 51.7 \frac{D_f}{3}. \quad (2.16)$$

The half-lives calculated by formulas (2.12–2.15) reflect properly the significant regularity of alpha-decays, namely the strong dependence on the energy of fragments W_{kA} (the decay energy).

The action of electromagnetic drivers on a substance with radioactive nuclei initiates the mechanism of excitation of low-frequency oscillations of unstable nuclei through the excitation of high-frequency oscillations of the surface, which is described in the following section.

The deformation of a nucleus and the appearance of its internal structure, as is seen from the above-presented relations, change essentially the lifetime of the nucleus. The spectrum of isomers arises. As a result of the appearance of an internal structure, the radioactive nuclei increase sharply their stability and become stable fractal isomers. A small part of nuclei decreases essentially (by 3-5 orders) the lifetime.

As an example, let us consider the decay of radium nuclei entering the composition of granite. The half-life of radium is equal to 1700 yr. Therefore, the measurements for months reveal no tendency for samples of granite to the diminution.

In Fig. 6, we present dependence (2.16) for a radium nucleus with a mass number of 226 and a charge number of 88 on the fractal dimension and the decay energy.

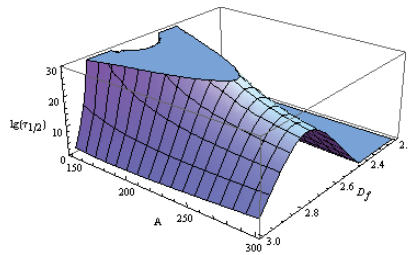


Fig. 6. Dependence of the logarithm of the half-life (in seconds) on the fractal dimension, decay energy (in MeV), and the mass number.

As the fractal dimension varies from 3 to 2.95 and as the binding energy increases, the half-life is reduced by 3 orders and becomes equal to 10^7 sec. Namely such values are measured in experiments (see Section 5.1).

2.3 Specific features of the evolution of the structure of a nucleus and the equations of dynamical harmonization

Of importance is the study of the mechanism of appearance of additional locally stable isomeric states. We will pay attention to the fact that the appearance of such type of states in nonlinear oscillatory systems is known for a long time and is related to the interaction of low- and high-frequency oscillations. Under certain conditions, when the eigenoscillations and a periodic external force interact, the effective potential is renormalized, and the damping arises [7].

The slow degree of freedom defining the process of deformation of a nucleus can be considered as a Brownian particle or an effective oscillator, whose dynamics is determined by a potential field and random actions from the side

of the rest, essentially faster, nucleonic degrees of freedom of a nucleus that is a heat bath with some temperature or the dispersion of random forces from the side of the bath.

The Lagrange function L_δ describing the effective oscillator related to the deformation of a nucleus can be presented in the form

$$L_0(\delta, \dot{\delta}) = \frac{1}{2} (A m_p) \dot{\delta}^2 - U_{def}(\delta). \quad (2.17)$$

The heat bath is the collection of a large number of noninteracting harmonic oscillators with coordinates $\alpha_{\lambda\mu}$, masses $D_{\lambda\mu}$, and frequencies $\omega_{\lambda\mu}$, which is described by the Lagrange function

$$L_{hb}(\alpha_{\lambda\mu}, \dot{\alpha}_{\lambda\mu}) = \sum_{\lambda\mu} \frac{D_{\lambda\mu}}{2} (\dot{\alpha}_{\lambda\mu}^2 - \omega_{\lambda\mu}^2 \alpha_{\lambda\mu}^2).$$

In the simplest case, the interaction between a particle and the heat bath is linear in the coordinates of oscillators, and the total Lagrange function takes the form

$$L(\delta, \dot{\delta}; \alpha_{\lambda\mu}, \dot{\alpha}_{\lambda\mu}) = L_0(\delta, \dot{\delta}) + L_{hb}(\alpha_{\lambda\mu}, \dot{\alpha}_{\lambda\mu}) + \sum_{\lambda\mu} g_{\lambda\mu}(\delta) \alpha_{\lambda\mu},$$

$$g_{\lambda\mu}(\delta) = \alpha_I C_{\lambda\mu}, \quad \alpha_I = \frac{1}{2} \left(\frac{\hbar^2 I^2}{2 Q_I} \frac{1}{\sigma_{surf} 4\pi R_A^2} \right), \quad (2.18)$$

where the functions $g_{\lambda\mu}(\delta)$ reflect the connection of oscillations of the mechanical and magnetic moments with the surface and are approximately written with regard for the stiffness of the nucleus surface, and the formula for $C_{\lambda\mu}$ is given in (2.15).

After the averaging over high-frequency oscillations of a thermostat (see, e.g., [10]), the large-scale force determining the oscillations can be given in terms of the renormalized potential

$$U_{eff}(\delta) = U_{def}(\delta) - \sum_i \frac{1}{2D_{\lambda\mu}\omega_i^2} (g_{\lambda\mu}(\delta))^2 \approx U_{def}(\delta) - \frac{1}{2C_{20}} g_{\lambda\mu}^2(\delta). \quad (2.19)$$

In this case, the friction force F_{frict} arises:

$$F_{frict} = -\gamma(\delta) \dot{\delta}; \quad \gamma(\delta) = \gamma_0 \left(\frac{dg_{\lambda\mu}(\delta)}{d\delta} \right)^2;$$

$$2\gamma_0 = \int_{-\infty}^{+\infty} dt \frac{1}{C_{\lambda\mu}} \cos \omega_i t \approx \int_{-\infty}^{+\infty} dt \frac{1}{C_{20}} \cos \omega_{20} t \approx \frac{\sqrt{2}}{C_{20}} \sqrt{\frac{D_{20}}{C_{20}}}. \quad (2.20)$$

After the averaging, the Langevin random force $F_L(r, t)$, which is determined by the parameters of the thermostat of high-frequency oscillations, takes the form

$$F_L(\delta, t) = \sqrt{D(\delta)} \Gamma(t); \quad \langle \varepsilon_{i0} \rangle = \frac{1}{2} k_B T;$$

$$D(\delta) = \gamma(\delta) k_B T = \left(\frac{dg_{\lambda\mu}(\delta)}{d\delta} \right) \gamma_0 k_B T. \quad (2.21)$$

Here,

$$\langle \varepsilon_{i0} \rangle = \frac{p_{i0}^2}{2D_{\lambda\mu}} + \frac{1}{2}D_{\lambda\mu}\omega_i^2 \langle q_{i0}^2 \rangle \approx \frac{p_{i0}^2}{2D_{20}} + \frac{1}{2}C_{20} \langle q_{i0}^2 \rangle = \frac{1}{2}k_B T$$

is the mean energy of oscillators from the thermostat of the surface modes of oscillations.

In this case, the equation for the deformation of a fractal nuclear isomer is the equation of a nonlinear oscillator with the potential energy $U_{eff}(\delta, \eta)$ and the damping $\gamma(\delta, \eta)$. Two last quantities depend slightly on the time through the order parameter $\eta(t)$ under the action of high-frequency forces $\xi(t) \approx \sum_i a_i \sin(\Omega_i t + \varphi_i)$ with the inhomogeneous slowly varying amplitude $\sqrt{D_0(E^2)D(\delta, \eta)}$ determined mainly by an external electromagnetic field. Thus, we have

$$\frac{d^2\delta}{dt^2} + \gamma(\delta, \eta) \frac{d\delta}{dt} + \frac{dU_{eff}(\delta, \eta)}{d\delta} = \sqrt{D_0(E^2)D(\delta, \eta)} \xi(t). \quad (2.22)$$

This equation involves the connection between the fractal dimension and the order parameter. Moreover, the effective potential of a low-frequency deformation $U_{eff}(\delta)$ involves the connection, which is determined by the evolution of the system to a quasistationary state, between a deformation of the nucleus and the order parameter $\delta(\eta)$.

The basic controlling parameter of the evolution of a nucleus is the order parameter, which is determined by the magnitude and the direction (sign) of the flow of entropic fields. The most powerful flows of entropic fields are created in systems with a fast controlled change of a structure as a result of bifurcations of the system and a change of its constraints near critical states or under the conditions of a generation of the longitudinal components of electromagnetic fields. The transverse components of electromagnetic fields ensure the excitation of high-frequency oscillations of the surface through the spin system of the nucleus and increase the dispersion of random forces.

The collective properties of nuclei are manifested, first of all, in the existence of oscillations of the surface of the nucleus near the spherically symmetric equilibrium shape and, for the deformed nuclei, in the rotation of a nucleus.

The nucleus can be rotated only around the axis perpendicular to the symmetry axis. In this case, the total angular momentum of the nucleus, \vec{J} , around this axis determines the discrete levels of the rotation energy:

$$W_J = \frac{\vec{J}^2}{2Q_{Inuc}} = \hbar^2 l(l+1) \frac{1}{2Q_{Inuc}}, \quad |\vec{J}| \approx \hbar l. \quad (2.23)$$

The inertia moment of the nucleus Q_{Inuc} depends on that which share of nucleons of the nucleus takes participation in the rotation. For example,

$$Q_{Inuc} = Q_{Solid} = \frac{2}{5}m_p r_0^2 A^{5/3}, \quad (2.24)$$

if the spherical nucleus is rotated as a solid; and

$$Q_{Inuc}(D_f, \varepsilon) = \frac{1}{3}m_p r_0^2 A \frac{2 + D_f}{D_f} (1 + \varepsilon)^2$$

for a fractal ellipsoid.

The oscillations of the nucleus surface around a spherically symmetric equilibrium shape are described by the collective variables $\alpha_{\lambda\mu}$ arising at the expansion of the distance $R(\vartheta, \varphi)$ from the coordinate origin to any point of the surface of the deformed nucleus in spherical harmonics [6]:

$$R(\vartheta, \varphi) = R_\eta(A) \left(1 + \sum_{\lambda, \mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\vartheta, \varphi) \right). \quad (2.25)$$

Here, $Y_{\lambda\mu}^*(\vartheta, \varphi)$ are spherical harmonics (the asterisk stands for the complex conjugation); ϑ and φ are, respectively, the polar and azimuthal angles of the surface point under consideration; $\lambda = 0, 1, 2, 3, \dots$, and μ can take values of $-\lambda, -\lambda + 1, \dots, \lambda - 1, \lambda$ at a fixed value of λ .

In this case, λ and μ set the oscillations of the nucleus with a definite angular momentum and the parity π :

$$\vec{\lambda}^2 = \hbar^2 l(l+1), \quad \lambda = \hbar\mu, \quad \pi = (-1)^\lambda. \quad (2.26)$$

The oscillations of the nucleus surface are described by the equations

$$\ddot{\alpha}_{\lambda\mu} + \omega_{\lambda\mu}^2 \alpha_{\lambda\mu} = 0. \quad (2.27)$$

The energy of these oscillations can be naturally expanded in a power series in the dynamical variables $\alpha_{\lambda\mu}$ and the conjugated velocities $\dot{\alpha}_{\lambda\mu}$. To within terms of the second order of smallness in $\alpha_{\lambda\mu}$ and $\dot{\alpha}_{\lambda\mu}$, this energy can be presented as

$$W = \sum_{\lambda\mu} \left(\frac{1}{2} D_{\lambda\mu} |\dot{\alpha}_{\lambda\mu}|^2 + \frac{1}{2} C_{\lambda\mu} |\alpha_{\lambda\mu}|^2 \right), \quad (2.28)$$

where $D_{\lambda\mu}$ is the mass parameter, and $C_{\lambda\mu}$ is a parameter characterizing the surface stiffness.

The quantized energy levels of these oscillation modes are given by the relation

$$W_n = \left(n + \frac{5}{2} \right) \hbar\omega_{\lambda\mu}, \quad \hbar\omega_{\lambda\mu} = \sqrt{C_{\lambda\mu}/D_{\lambda\mu}}, \quad (2.29)$$

where n is the total number of phonons (i.e., quanta of oscillations of the nucleus surface), $\hbar\omega_{\lambda\mu}$ is the energy of a phonon, and the mass parameter and the stiffness parameter are determined by the relations

$$\begin{aligned} D_{\lambda\mu} &= \frac{3}{4\pi} \frac{1}{\lambda} A m R^2 = \frac{1}{\lambda} \frac{3}{4\pi} m_p r_0^2 A^{1+2/D_f}; \\ C_{\lambda\mu} &= \frac{1}{4\pi} (\lambda - 1)(\lambda + 2) \sigma_{surf} R_\eta^2 - \frac{3}{2\pi} \frac{\lambda - 1}{2\lambda + 1} \frac{e^2 Z(Z - 1)}{R_\eta}. \end{aligned} \quad (2.30)$$

The main contribution to the oscillations is given by the mode with $\lambda = 2$. In this case, relation (2. 28) describes the energy of a harmonic oscillator with five degrees of freedom: $\mu = -2, -1, 0, 1, 2$. The frequencies of oscillations of the nucleus surface (2/29) take values in the range of gamma-radiation. The oscillations with $\lambda \gg 2$ are a high-frequency “ripple” on the deformed surface of the nucleus, which can be considered as a heat “bath” containing also the adiabatic lower-in-frequency oscillations of a shape of the ellipsoidal nucleus.

For many heavy nuclei, the approximation based on the value of inertia moment of a solid leads to large deviations from experimental data. This is related to the influence of the internal structure of a nucleus. Let us assume that the rotation is created only due to the propagation of a deformational wave in the incompressible nuclear fluid participating in the vortex-free hydrodynamic motion. Then the inertia moment Q_{IG} will be less than that of a solid by the squared parameter of quadrupole deformation [6]

$$Q_{IG} = 3D_{2\mu}\beta^2 = Q_{Solid}\varepsilon^2, \quad (2.31)$$

where $\beta = \sum_{\mu} |\alpha_{2\mu}|^2$ is the parameter of quadrupole deformation, and $D_{2\mu}$ is the mass parameter for the quadrupole mode of a vortex-free flow of the fluid. The disagreement with experiment under these assumptions somewhat decreases, and the use of hydrodynamic models improves the fitting of experimental data as compared with the drop model. However, in order to match the experimental data with the hydrodynamical model, it is necessary to use large form-factors for heavy nuclei [8]. It is easy to see that the appearance of a fractal structure of heavy nuclei leads to a better agreement with experiment, since it significantly changes the coupling constants in the desired direction:

$$\frac{D_{\eta}}{D_G} \approx A^{\frac{2}{3}} \frac{(3 - D_f)}{D_f}. \quad (2.32)$$

In Fig. 8, we present the dependence of the ratio of the mass parameters of a nucleus in the model of fractal structure and in the hydrodynamical model on the order parameter.

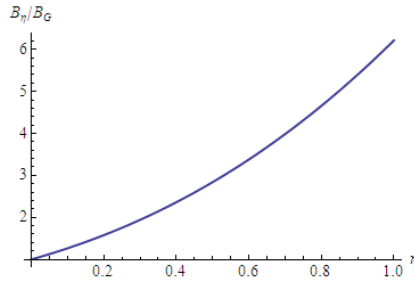


Fig. 8. Dependence of the ratio of the mass parameters of nucleus ^{240}Pu calculated in the model of fractal structure and in the hydrodynamical approximation on the order parameter.

It is seen that, as the order parameter and, hence, a deviation of the fractal dimension of a nucleus from 3 increase, the rated mass parameter of ^{240}Pu grows, by allowing us to fit the experimental data.

The internal structure of a nucleus revealing itself in the spatial density distribution of the nuclear matter affects essentially the splitting of the energy levels due to the spin-orbit interaction:

$$\begin{aligned}
V_{ls} &= f(r) \vec{l} \vec{s}, & f(r) &\propto \frac{1}{r} \frac{\partial \rho_{nuc}(r)}{\partial r}, \\
\vec{l} \vec{s} &= \begin{cases} \hbar^2 (l/2), & j = l + 1/2 \\ -\hbar^2 (l+1)/2, & j = l - 1/2 \end{cases}, \\
\vec{j}^2 &= (\vec{l} + \vec{s})^2 = l^2 + 2\vec{l} \vec{s} + s^2.
\end{aligned} \tag{2.33}$$

The spin-orbit splitting of the level nl is

$$W_{nl(j=l+1/2)} - W_{nl(j=l-1/2)} = C_{ls} (l + 1/2), \quad C_{ls} = -\hbar^2 \bar{f}, \tag{2.34}$$

where \bar{f} is the mean value of the function $f(r)$. The spin-orbit splitting increases with the orbital moment l . The evaluation of the constant C_{ls} gives

$$C_{ls} \approx 20A^{-2/D_f} \text{MeV}. \tag{2.35}$$

It is seen that the constant C_{ls} can be significantly decreased proportionally to $\approx A^{-1/3}$ for a fractal cluster, as the fractal dimension approaches the limit $D_f \approx 2$ corresponding to bubble nuclei and a power decrease in the density. In this case, the distance between the energy levels of a cluster is decreased as well.

The oscillations with $\lambda \gg 2$ are a high-frequency ‘‘ripple’’ on the deformed surface of a nucleus, which can be considered as a heat ‘‘bath’’ for the adiabatic oscillations of the ellipsoid representing the shape of the nucleus. The temperature of this bath can be changed due to the dissipation of the energy of oscillations of spins in an external electromagnetic field $E(\omega)$ with frequency ω and in a magnetic field $H(t)$.

The magnetic field determines the eigenfrequencies of rotation of nuclear spins $\Omega_m(H)$:

$$\begin{aligned}
\Omega_m(H) &= (2m + 1) \mu_N H \approx (2m + 1) 4.4107 \left(\frac{H}{10^4} \right), \\
\mu_N &= \frac{e\hbar}{2m_p c} \approx 3.1524 \cdot 10^{-18} \text{MeV/Gauss},
\end{aligned} \tag{2.36}$$

which are less than the frequencies of oscillations of the surface $\omega_{\lambda\mu}$ by many orders, and the absorbed energy of an electromagnetic wave with frequency ω has a resonance character:

$$P_\Omega(t, \omega) = \sigma_0 \frac{1/\tau_{eff}}{(\omega \pm \Omega_m(H(t)))^2 + (1/\tau_{eff})^2} E^2,$$

where τ_{eff} is the efficient dissipation time. By averaging the energy of the electromagnetic field absorbed by surface oscillations over frequencies, we obtain

$$\begin{aligned}
P_\Omega(t) &= \sum_m \sigma_0 \int \frac{1/\tau_{eff}}{(\omega \pm \Omega_m(H(t)))^2 + (1/\tau_{eff})^2} |E(\omega)|^2 d\omega \\
&\approx \sum_m \sigma_0 \tau_{eff} |E(\Omega_m(H(t)))|^2.
\end{aligned} \tag{2.37}$$

Thus, the temperature of chaotic surface oscillations of the nucleus turns out proportional to the dissipation energy:

$$T \propto P_{\Omega}(t). \quad (2.38)$$

In correspondence with the principle of dynamical harmonization, the changes of a structure in nuclei and the appearance of the most probable states of nuclei are initiated by the coherent accelerations corresponding to the entropic fields $S = a_s \sin(\omega_s t + \varphi_s)$ and the electromagnetic fields $E = E_0 \sin(\omega_{ext} t + \varphi_{ext})$ with relatively low frequencies.

The evolution of the structure of an isomer is defined by the principle of dynamical harmonization of systems with varying constraints [1, 4] and the entropic forces, which can be expressed through the entropy gradient fields:

$$F_S \propto \nabla S, \quad U_{str}(\eta) = \int_0^{\eta} \nabla S d\eta. \quad (2.39)$$

The analysis of a dynamics of systems with varying constraints indicates the explosive growth of the structure inertia with the order parameter (see [1–3]), which can be approximated by the formula:

$$m_{str}(\eta) = \frac{D_{20}}{(1 - 2\eta)^{\gamma_{str}}}$$

or

$$m_{str}(D_f) = \frac{\frac{3}{8\pi} m_p r_0^2}{\left(1 - 2\frac{3 - D_f}{D_f - 1}\right)^{\gamma_{str}}} A^{1 + \frac{2}{D_f}}, \quad (2.40)$$

where the mass parameter for low-frequency deformational oscillations

$$D_{20} = \frac{3}{8\pi} m_p r_0^2 A^{1 + \frac{2}{D_f}},$$

and $\gamma_{str} \approx 1.83$.

With regard for the dependence of the order parameter on the fractal dimension (2.2), we obtain the second-order equation. It is convenient to present this equation in the form of a system:

$$\dot{D}_f = u_{D_f}, \quad (2.41)$$

$$\dot{u}_{D_f} = \alpha(D_f, T, \nabla S) D_f + \beta(D_f, u_{D_f}) u_{D_f} + \Pi(D_f, u_{D_f}). \quad (2.42)$$

It is of importance that these equations involve a discrete sequence of locally stable fractal isomeric states corresponding to a discrete collection of the order parameters, fractal dimensions, deformations, and binding energies.

We can show that the nuclear dynamics defined by Eqs. (2.22, 2.41–2.42) with a high nonlinearity in the conditions for the external forces to be nonstationary has a dimension of more than 3 in the phase space and is characterized by the dynamical chaos related to the presence of several attractors.

In Fig. 9, we show the dependence of the potential energy entering the equation of harmonization on the fractal dimension and the deformation. As is seen, three potential “wells” (three attractors) and the domains of unstable equilibrium are present.

At the every current moment of the macroscopic time, the nuclear system have time to pass in one of the collection of stable states corresponding to the current values of coherent acceleration and temperature. Since each nucleus starts to evolve from some definite internal states and the values of external parameters somewhat different from those for other nuclei, there appears a statistical ensemble with a certain distribution over internal structures, shapes, and appropriate binding energies. The distribution of nuclei over collective parameters evolves slowly in the course of the macroscopic time. Moreover, this evolution reveals the following essential specific feature.

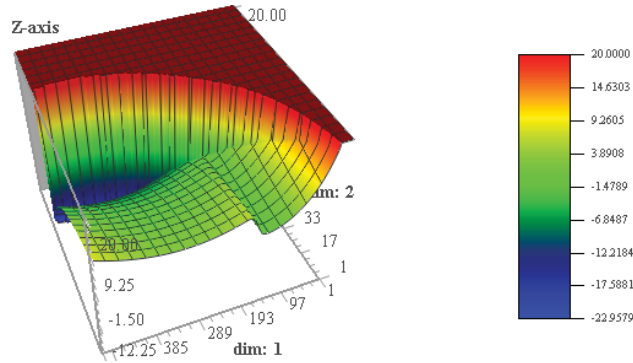


Fig. 9. Surface representing the potential energy of the system in the fractal dimension – deformation of a nucleus coordinates. Three stable domains of parameters and one domain of unstable equilibrium are seen.

The dependence of the structure mass on the order parameter or the fractal dimension ensures the phenomenon of hysteresis at the formation of structures. Under the action of forces with a positive acceleration some structures are formed from the initial state with some value of the order parameter. In this case, the order parameter increases, by attaining the relevant maximal value.

At the cyclic action of mass forces on the system, the acceleration changes the sign on the final stage of the process. If the system would be linear, and its structural inertia would not depend on the order parameter, then the system would return in the initial state after the termination of the action of mass forces. However, since the structural inertia increase with the order parameter, the evolution of the system does not return it in the initial state at a decrease of the forces, but ensures the “residual” value of order parameter, which differs from the initial value and is determined by the evolution under the action of a mass entropic force.

In other words, after the termination of the action of an external entropic force breaking the symmetry of the system, the system acquires a long-range order characterized by the collective variables and the parameter η . This long-range order determines the final distribution of nuclei over all collective variables. The subsequent evolution of the system occurs in correspondence with this long-range order and the various degrees of the stability of components of the system relative to nuclear processes.

3 Beta-processes at the evolution of a structure of nuclei under the action of mass forces

3.1 Probability of beta-processes for fractal isomers

The possibility of the appearance of nucleonic structures in a nucleus leads to the significant change of running nuclear processes. Let us consider beta-processes with regard for the structure of a nucleus.

By the Fermi theory, the characteristic half-life of nuclei τ_β and the probability of their decay P_β depend on the binding energy of a nucleus and its internal structure:

$$P_\beta = \frac{\log(2)}{\tau_\beta} \approx \frac{g^2}{4\pi^3} \frac{m_e^5 c^4}{\hbar^7} \Phi_0(\varepsilon_e) = k_\beta \Phi_0(\varepsilon_e), \quad k_\beta = 5598.41. \quad (3.1)$$

Here, $g = 0.88 \cdot 10^{-4} \text{MeV} \cdot \text{fm}^3$ is the weak interaction intensity, and the function $\Phi_0(\varepsilon_e)$ determines the influence of peculiarities of the structure of a nucleus on the decay probability:

$$\Phi_0(\varepsilon_e) = \frac{\varepsilon_e}{2} \log(\varepsilon_e + \sqrt{\varepsilon_e^2 - 1}) + \frac{1}{30} (2\varepsilon_e^4 - 9\varepsilon_e^2 - 8) \sqrt{\varepsilon_e^2 - 1}. \quad (3.2)$$

The parameter ε_e is the amount of the energy released in the decay referred to the intrinsic energy of an electron. With regard for the possibility to change the structure of a nucleus, this quantity is given by the function

$$\begin{aligned} \varepsilon_e &= \frac{1}{m_e c^2} (m_n c^2 - m_p c^2 - m_e c^2 + B(A, Z+1, D_f) - B(A, Z, D_f)) \\ &= 1.53032 + \frac{B(A, Z+1, D_f) - B(A, Z, D_f)}{m_e c^2}. \end{aligned} \quad (3.3)$$

It is convenient to write the final relation for the decay probability, by separating the factor P_β^0 corresponding to the probability of the process of decay of the nucleus without regard for the possibility to change its internal structure:

$$P_\beta = P_\beta^0 \chi(A, Z, D_f, D_{f0}), \quad \chi(A, Z, D_f, D_{f0}) = \frac{\Phi_0(\varepsilon_e(A, Z, D_f))}{\Phi_0(\varepsilon_e(A, Z, 3))}. \quad (3.4)$$

The probability of the beta-process depends very essentially on the binding energy of products of the reaction. The higher the binding energy of products relative to the binding energy of the initial nucleus, the faster the beta-process will run. We illustrate the decay of nuclei with regard for the structure by the example of nucleus $^{90}\text{Sr}_{38}$. In Fig. 10, we give the plot of the dependence of the nucleus charge, which ensures the beta-stability for a nucleus with a mass number of 90, on the fractal dimension.

It is seen from Fig. 10 that nucleus $^{90}\text{Sr}_{38}$ with $Z = 38$ is unstable at a fractal dimension equal to 3 and can be stable at the transition in the nucleus with $Z = 39$. In Fig. 11, we present the dependence of the specific binding energy per nucleon of a nucleus with a mass number of 90 on the charge number.

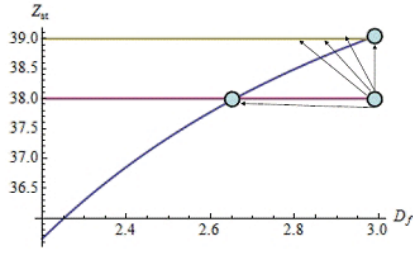


Fig. 10. Dependence of the charge of a stable nucleus with a mass of 90 on the fractal dimension.

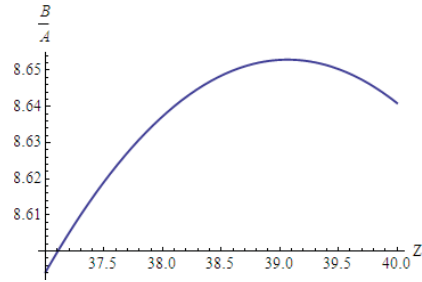


Fig. 11. Dependence of the specific binding energy of a nuclear drop with a mass of 90 on the nucleus charge.

It is seen that the beta-process transfers the nucleus in the state with maximal binding energy, if the nucleus charge increases from 38 to 39. However, the presence of a degree of freedom related to the structure makes another process possible: an increase in the stability of a nucleus due to a decrease of the fractal dimension. As the dimension $D_f = 2.67$, nucleus ${}^{90}\text{Sr}_{38}^*$ turns out stable.

In Fig. 12, we present the dependence of the difference of the specific binding energies of adjacent fractal nuclei (their charges differ by 1) with a mass number of 90 on the fractal dimension.

It is seen that only the nuclei with the fractal dimension $D_f > D_{fb} \approx 2.8$ participate in the process of beta-decay. For them, this process is energy-gained. The process with a decrease in the charge is unfavorable for ordinary nuclei ${}^{90}\text{Sr}$. However, it would run with a high probability in bubble and quasibubble nuclei ${}^{90}\text{Sr}$ (for $2.1 < D_f < 2.53$).

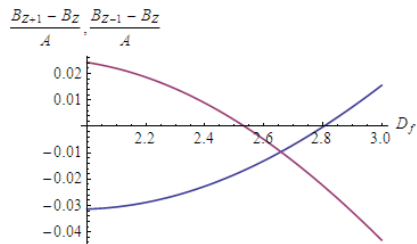


Fig. 12. Difference of specific binding energies for adjacent nuclei with an identical mass number of 90 as a function of the fractal dimension of nuclei. The blue and red curves correspond, respectively, to the increase and the decrease of the nucleus charge by 1.

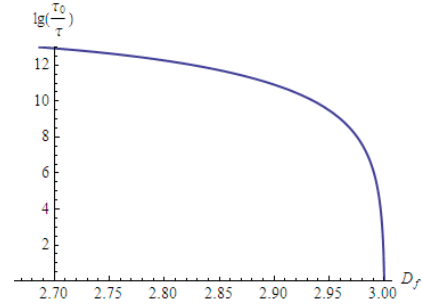


Fig. 13. Common logarithm of the ratio of the half-life of a nuclear drop with mass 90 and charge 38 to the half-life of a nucleus with internal structure as a function of the fractal dimension of the nucleus.

A variation of the structure of a nucleus changes the specific binding energy of ${}^{90}\text{Sr}_{38}^*$ from 0 to 18 MeV per nucleon and the probability of beta-decay by many orders (see Fig. 13), so that the decrease of the fractal dimension can decrease the half-life by 13 orders.

3.2 Regularities of the decays of nuclear systems with regard for their coherence

We denote the probability of decay of a nuclear system in a quasistationary state n by p_n . Then, for a large number of nuclei in the state n , the law of the time variation of the number of nuclei decaying independently is given by the relation

$$\frac{dN}{dt} = -\omega N(t), \quad \omega = p_n = \frac{\log 2}{\tau_n},$$

where τ_n is the half-life of the n -th state. A complicated structure of the surface of the nuclear potential leads to a possibility of the existence of quasistationary states, whose half-lives and their reciprocal values (frequencies) are harmonics and subharmonics of the basic frequency ω in the range

$$\frac{\omega}{b^{N_1}} < \omega_n < \omega b^{N_2}, \quad b \geq 2, \quad (3.5)$$

and determine a fractal time structure of the process of decay with some dimension $1 \leq D_f \leq 2$. Such dependence can be qualitatively presented with the use of a wide-band fractal Weierstrass function

$$\begin{aligned} W(t, D_f) = \varepsilon & \frac{\sqrt{2}(1 - b^{2(D_f-2)})^{1/2}}{(b^{2(D_f-2)N_1} - b^{2(D_f-2)(N_2+1)})^{1/2}} \\ & \times \sum_{n=-N_1}^{N_2} \frac{1}{b^{(2-D_f)n}} \cos(2\pi\omega b^n t + \vartheta_n). \end{aligned} \quad (3.6)$$

The decay of a nuclear system with a fractal structure must occur on all time scales and satisfy the equation

$$\begin{aligned} \frac{dN}{dt} &= -\gamma_n(t) N(t), \\ \gamma_n &= p_n W(t, D_f), \end{aligned} \quad (3.7)$$

Then the number of radioactive nuclei and the activity are described as follows:

$$\begin{aligned} N(t) &= N_0 \exp\left(-p_n \int_0^t W(t, D_f) dt\right), \\ A(t) &= -\frac{dN}{dt} = p_n W(t, D_f) N_0 \exp\left(-p_n \int_0^t W(t, D_f) dt\right). \end{aligned} \quad (3.8)$$

The plots of the number of nucleons and the activity are given in Figs. 14.

We may conclude that the high-frequency oscillations of the activities of isomers are related to the circumstance that the nuclear dynamics has clearly pronounced properties of a complicated nonlinear system with dynamical chaos.

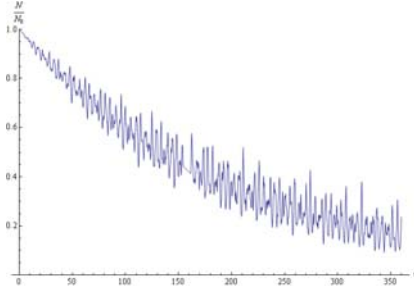


Fig. 14 a). Relative number of radioactive nuclei versus the time.

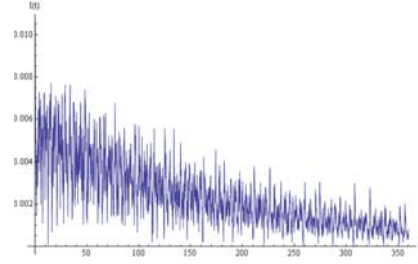


Fig. 14 b). Activity of decays in relative units versus the time.

3.3 Evolution of the activities in a mixture of fractal isomers

By the properly calculated actions on the spin system of a nucleus, it is possible to direct the energy of chaotic oscillations of the nucleus surface onto a slow deformation of the intercomponent potential in a nuclear quasimolecule for the formation of an isomer with the necessary reserve of stability, i.e., with the necessary depth of a self-consistent “pit” on the curve of the interaction potential for formed parts (subsystems) of the isomer.

Qualitatively, the spectrum of nuclei after the treatment of a radioactive isotope (${}_{38}\text{Sr}^{90}$, in this case) can be presented by the following groups:

- Invariable nuclei ${}^{90}\text{Sr}_{38}$;
- Nuclei with the fractal dimension $D_f < D_{fb} = 2.8$, which are quasistationary fractal isomers
- Nuclei with the fractal dimension $D_{fb} < D_f < 3$, which are fractal isomers with the half-life significantly less than that of ${}_{38}\text{Sr}^{90}$.

The number of isomers formed in the course of the action is determined by their function of distribution over fractal dimensions. Let us estimate the distribution function in view of the counteraction of two tendencies, in a very rough approximation, in the process under study. They are the inclination of the system to obtain the energy gain due to a change of the fractal dimension and to decrease the necessary expenditures of energy on a deformation of the nucleus. These two tendencies can be described, by introducing the effective potential energy depending on the fractal dimension and characterizing the energy “cost” of the motion along the evolution path,

$$U_{eff}(A, Z, D_f) \approx \frac{B(A, Z, D_{f_{opt}}) - B(A, Z, D_f)}{A} + k_{def} \left(\frac{\Delta R(A, D_f)}{R} \right)^2. \quad (3.9)$$

In this relation, $k_{def} \approx \frac{\sigma_{surf}}{\pi} R^2$ is the coefficient of stiffness of the nucleus, the first term characterizes the energy gain due to the displacement to the optimal value of fractal dimension, and the second term gives a simple estimate of the energy loss at an elastic deformation of the nucleus in such

displacement. In view of the relation $R \approx 1.2A^{1/D_f}$, we obtain

$$\left| \frac{\Delta R(A, D_f)}{R} \right| \approx \log(A) \frac{|\Delta D_f|}{D_f^2}. \quad (3.10)$$

For a Sr nucleus, $\Delta R \approx 2.69 |\Delta D_f|$ in fm.

In the general case, the distribution function of isomers has power asymptotics, whose exponent is determined by the parameter q ,

$$f(D_f) \approx \exp_q(-U_{eff}(A, Z, D_f)/T_{eff}), \quad (3.11)$$

and takes the form shown in Fig. 15.

The parameters q and T_{eff} depend on the characteristics of a used driver of the evolution of nuclei. Their variation causes changes of the distribution function of products and, on the whole, the efficiency of the process of neutralization.

In correspondence with the distribution function, the share of stable isomers in the system is of the order of $p_{st} \approx \int_{2.67}^{2.8} f(D_f) dD_f$. For the functions shown in Fig. 15, $p_{st} \approx 0.75$.

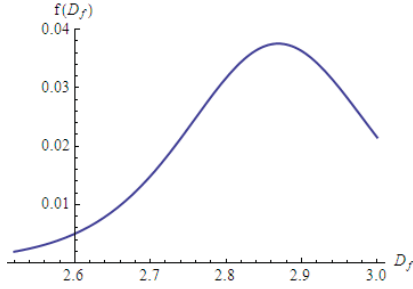


Fig. 15 a). The distribution function of isomers versus the fractal dimension. The exponent of the asymptotic is determined by the intensity and the quality of the action.

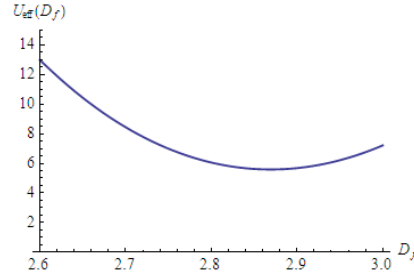


Fig. 15 b). Dependence of the effective potential energy on the fractal dimension. The minimum corresponds to the optimal ratio of the deformation energy and the binding energy with regard for a change of the structure of isomers.

The decay intensity can be approximately presented by a sum of exponentially decreasing components with the strongly different half-lives:

$$\begin{aligned} A(t) &\approx \frac{N_0}{\tau_{eff}} \frac{1}{(1 + t/\tau_{eff})^{2-\nu}} \\ &\approx \frac{0.693N_1}{\tau_1} \exp\left(-0.693 \frac{t}{\tau_1}\right) + \frac{0.693N_2}{\tau_2} \exp\left(-0.693 \frac{t}{\tau_2}\right), \\ \tau_1 &\ll \tau_2, \quad N_1 \ll N_2. \end{aligned} \quad (3.12)$$

Below, we will give additionally some details of the evolution, which are essential on the initial stage and for the organization of a sequence of actions on the nuclear system separated by some characteristic time interval, which will be estimated.

The action of a wide-band radiation on a system of nuclei and the mass forces induced by it lead to the appearance of the sources of fractal nuclear isomers with various intensities Q_i . The intensities are equal to the number of corresponding fractal isomers created for one second under the action of mass forces. The intensities can be presented in the form of a product of the probability of the process $p_i \approx \frac{0.693}{\tau_{tr}}$, and the initial number of Sr nuclei $Q_i = p_i N_{i0}$. Here, τ_{tr} is the characteristic time of formation of isomers.

As a result of the action of these sources, the number of corresponding isomers must increase linearly with the time, $N_i(t) = Q_i t$. However, if we consider the boundedness of their lifetimes, i.e., the decay of these isomers with characteristic half-lives τ_i , then the time dependences of the number of isomers and their activities take the form

$$\begin{aligned} N_i(t) &= Q_i t \exp\left(-0.693 \frac{t}{\tau_i}\right), \\ A_i(t) &= Q_i \left| 0.693 \frac{t}{\tau_i} - 1 \right| \exp\left(-0.693 \frac{t}{\tau_i}\right). \end{aligned} \quad (3.13)$$

We indicate a peculiarity of these dependences such that, when the exponential mode is approached, the activity maximum position corresponds to $t_{\max} = 2.88 \tau_i$, and the maximal value of activity $A_{\max} = 0.135 Q_i$.

We now introduce an approximate model of the process, by replacing the full spectrum of products by the following 5 components:

- Stable nuclei with the fractal dimension in the interval $2.67 < D_f < 2.8$; the source power Q_0 .
- First fraction of rapidly decaying nuclei with the fractal dimension $2.8 < D_f < 2.9966$; the source power Q_1 with the mean half-life $\tau_1 \approx 82\,000$, i.e., up to one day.
- Second fraction of rapidly decaying nuclei with the fractal dimension $2.9966 < D_f < 2.9986$; the source power Q_2 with the mean half-life $\tau_2 \approx 2.58 \cdot 10^6$, i.e., up to one month.
- Comparatively slowly decaying fraction with the fractal dimension $2.9986 < D_f < 3$; the source power Q_3 with the mean half-life $\tau_3 \approx 9.1 \cdot 10^7$, i.e., up to three years.
- Part p_0 of initial radioactive Sr nuclei not undergoing the action.

Let us write the numbers of nuclei in the corresponding components:

$$\begin{aligned} N_0(t) &= Q_0 t, & Q_0 &= p_0 N_{i0}, \\ N_1(t) &= Q_1 t \exp(-0.693 t / \tau_1), & Q_1 &= p_1 N_{i0}, \\ N_2(t) &= Q_2 t \exp(-0.693 t / \tau_2), & Q_2 &= p_2 N_{i0}, \\ N_3(t) &= Q_3 t \exp(-0.693 t / \tau_3), & Q_3 &= p_3 N_{i0}, \\ N_i(t) &= Q_i \exp(-0.693 t / \tau_i), & Q_i &= p_i N_{i0}, \\ & (N_{i0} - (Q_0 + Q_1 + Q_2 + Q_3) t), \\ p_i &= 1 - \frac{Q_0 + Q_1 + Q_2 + Q_3}{N_{i0}}. \end{aligned}$$

The probabilities of processes are calculated with regard for the function of distribution of isomers over the fractal dimensions. In this case, the number

of active nuclei $N_a(t)$ and their total activity vary by the laws:

$$N_a(t) = N_i(t) + N_1(t) + N_2(t) + N_3(t);$$

$$A_a(t) = \left| \frac{d}{dt} (N_i(t) + N_1(t) + N_2(t) + N_3(t)) \right|.$$

Since the activity of isomers is inversely proportional to the half-life of a corresponding component, the main contribution to the estimates by these formulas is given by the fastest component from the spectrum of isomeric products after the action.

4 The model based on a system of iterated functions (SIF) for the dynamics of a system of many nuclei under the action of mass forces

In the qualitative and quantitative description of the long-term evolution of a nuclear system, several components of the arisen distribution of fractal isomers are of great importance:

- Stable nuclei with a large reserve of stability;
- Stable with a small reserve of stability;
- Groups of nuclei, being in unstable states with a definite lifetime.

It is worth noting that:

- Most important parameters of fractal clusters are: A , Z , D_f , and δ . In this case, we can reduce the number of parameters, by using the dependence of a deformation of the fractal cluster on the order parameter (or the fractal dimension);
- By virtue of the nonlinearity of low-frequency deformational oscillations of the nucleus and the presence of high-frequency inhomogeneous oscillations of the surface, a discrete sequence of quasistationary levels of the structure of fractal clusters appears with the corresponding sequence of the values of fractal dimension $D_f(k_f)$, which correspond to new quantum numbers of the structure $k_f = 1, 2, 3, \dots$;
- Since the characteristic times of the nuclear dynamics with regard for Eqs. (4.45) are less by many orders than the characteristic times of the action of mass entropic forces controlling the process, the general dynamics of the system can be most adequately represented with the use of the apparatus of mappings in the three-dimensional space of parameters (A, Z, D_f) ;
- Under real conditions, the system is statistical and consists of a large number of nuclei.

It is clear from (2.41–2.42) that the dynamics of a stable nuclear system, for which A and Z are invariable, is described by a two-dimensional mapping on the phase plane (D_f, u_f) ,

$$\begin{pmatrix} (D_f)_{i+1} \\ (u_f)_{i+1} \end{pmatrix} = \begin{pmatrix} a_{11}^j & a_{12}^j \\ a_{21}^j & a_{22}^j \end{pmatrix} \begin{pmatrix} (D_f)_i \\ (u_f)_i \end{pmatrix} + \begin{pmatrix} b^j \\ c^j \end{pmatrix},$$

$$M^j = \begin{pmatrix} a_{11}^j & a_{12}^j \\ a_{21}^j & a_{22}^j \end{pmatrix}, \quad V^j = \begin{pmatrix} b^j \\ c^j \end{pmatrix}, \quad (4.1)$$

which is set by the two-dimensional matrix M^j and the vector V^j . In the general case, these quantities vary from one time point to another one, since they depend on the current value of fractal dimension. We may consider that these pairs of matrices and vectors form a discrete set, whose elements are numbered by the index j .

This scheme has two limiting cases:

- Matrices and vectors are independent of the current coordinates. In this case, the ordinary mapping in the phase space, which is valid only for the equations with constant coefficients, is realized. But it is not true in the case of the evolution;
- Matrices and vectors are determined at each step exactly by Eq. (2.41–2.42). In this case, relation (4.51) realizes a discrete scheme of solution of a nonlinear equation for a single nucleus.

However, in this discrete scheme, we can try to use simultaneously the equation of harmonization (by involving the collective properties of a nucleus) and the statistical properties of the great number of nuclei (by approaching the description of real physical processes).

4.1 Statistical properties of a system of nuclei and the model based on SIF

The statistical properties of an open system of nuclei under conditions of the actions of electric and entropic fields are manifested in that the fast evolution of each nucleus by Eq. (4.1) leads to the appearance of a quasistationary probability distribution function $p(D_f)$ over the fractal dimensions of nuclei. We will use a discrete representation of this distribution function in the form of a multidimensional vector:

$$p^j = p\left((D_f)^j\right), \quad j = 1, 2, 3, \dots, N_j. \quad (4.2)$$

Here, the whole range of variation of the fractal dimension from 2 to 3 is divided into N_j segments $1/N_j$ in length, and the values of fractal dimension in the middle of each of them are $(D_f)^j$. For each of these segments, we determine the collection of matrices M^j and the vector V^j by the equation of dynamical harmonization. It is natural that these collections have the same distribution function p^j , i.e., the probability of appearance of these mappings at the evolution of the set of nuclei under consideration.

It is clear that, in order that the final state exist, the operators must satisfy the requirement that the Jacobians of all matrices be less than 1. This requirement for the given initial nuclear system is the necessary limitation on the parameters of drivers.

The construction arisen in the above-described way is called a system of iterated functions (SIF, see, e.g., [9]) in mathematics and is a powerful tool of the theory of fractals, by occupying an intermediate place in the description of dynamical systems between mappings and differential equations.

By definition, SIF is a set of affine transformations $T := \{T_j|_{j=1, \dots, N_j}\}$ that map a subset Ω of the d -dimensional real space into itself:

$$T^j(r) = M^j r + V^j, \quad (4.3)$$

where $r \in \Omega$ is a d -dimensional vector, M^j is some definite square matrix (with a dimension of $d \times d$) and V^j is a definite d -dimensional vector. We also require that the transformations be contractive, i.e., that the Jacobian $J^j = |\det M^j|$ be less than 1. Each transformation corresponds to the vector of probabilities p^j , so that $\sum_{j=1}^{N_j} p^j = 1$.

In the general case, the fractal collective dynamics of the system in a space Ω with any fixed dimension is described with the help of the relevant SIF in the following way:

1. We choose a point $r_0 \in \Omega$ and, by choosing a transformation

$$T^j r = M^j r + V^j$$

with probability p^j , calculate $r_1 = T^j(r_0)$.

2. By iterating this procedure, we generate a new point $r_i = T^{j(i)}(r_{i-1})$ in Ω on each step.

The function $T^{j(i)}$ is chosen in the random manner in correspondence with the given probabilistic vector $p^j = (p_1, p_2, \dots, p_{N_j})$ from N_j transformations T^j .

The ordinary discrete transformation $f : \Omega \rightarrow \Omega$ is the limiting case of SIF with the degenerate vector of probabilities of the transformations $p = (p_k = 1, p_j = 0, j \neq k)$. It was proved [9] that the sequence r_0, r_1, r_2, \dots is attracted to a zero-volume subset $F \subset \Omega$, which has, in the general case, a fractal structure.

The fractal structure of this limiting set is determined only by a collection of linear transformations, which depend on the properties of input nuclei and on the parameters of external electromagnetic and entropic drivers. The influence of the nontrivial vector of probabilities of operators I reduced only to a small effect on the rate of a transient process, rather than on the final result.

By another theorem proved in the theory of SIF, there exists a collection of linear operators for any attractor in the input phase space that transfer the system into a required state with a quadratic rate. In other words, any final state of a nuclear system admits the corresponding electromagnetic and entropic control.

The evolution of a nuclear system described above within the formalism of SIF was realized under the assumption of the constancy of mass and charge numbers of nuclei, i.e., we considered the evolution of nuclear isomers. If the evolution is considered with possible changes of mass and charge numbers, the dimension of the space Ω should be increased so that the products of nuclear processes (e.g., the quantities A and Z should be included as the coordinates of the space) will be included. The vectors of state of a nuclear system (like the matrices and the vectors of mappings) become four-dimensional. Moreover, the number of mappings increases even in the case where the mass and charge numbers are changed by 1, which corresponds to the conservation of mass and charge numbers or to their increase or decrease due to the beta-processes and

the processes of transfer of one neutron from nucleus to nucleus. With regard for these all processes, the number of mappings increases by nine times.

Among these processes, the processes of capture of a neutron with the conservation of a charge are the simplest. In this case, the space of states is three-dimensional, and the number of mappings increases only by three times. Moreover, the matrices of all mappings in the extended space can be written in the form

$$M^j = \begin{pmatrix} 1 & \tau & 0 \\ \alpha_A ((D_f)_i, \nabla S) & (1 - \beta\tau) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.4)$$

and the vector of shift for each matrix can be in one of three forms

$$V^j = \begin{pmatrix} 0 \\ \Pi_A^* (Z, (D_f)_j) \tau \\ 0 \end{pmatrix}, \quad V^j = \begin{pmatrix} 0 \\ \Pi_A^* (Z, (D_f)_j) \tau \\ 1 \end{pmatrix},$$

$$V^j = \begin{pmatrix} 0 \\ \Pi_A^* (Z, (D_f)_j) \tau \\ -1 \end{pmatrix}.$$

The probabilities of these mappings are determined in correspondence with the probabilities of the nuclear processes of capture of neutrons.

The successive application of mappings, i.e., the realization of the dynamics of a nuclear system in correspondence with the obtained SIF, leads to the manifestation of a structure in the space of states that is determined by the initial nuclear system and the controlling actions of electromagnetic and entropic fields. The structures in the space of states demonstrate the evolution of the distribution of the nuclear system over its basic parameters.

We now describe the direct means of reconstruction of SIF by an experimental realization x_i . With the help of the operator of regularization, we recover a trajectory on the plane $(\langle x_i \rangle, \frac{d}{dt} \langle x_i \rangle)$, where the regularization of a series is performed with the help of a fractional integral. The trajectory on this phase plane can be represented with the help of the contractive mappings and the vectors of shift in correspondence with the definition of SIF (4.29):

$$\begin{pmatrix} \langle x_{i+1} \rangle \\ \frac{d \langle x_{i+1} \rangle}{dt} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \langle x_{i+1} \rangle \\ \frac{d \langle x_{i+1} \rangle}{dt} \end{pmatrix} + \begin{pmatrix} b_i \\ c_i \end{pmatrix}. \quad (4.5)$$

Having determined the elements of the matrix and the vector at every point on the plane, we find the probabilities of appearance of each of the mappings. As an example, we consider the sequence of points that are generated by a trajectory on the attractor of a nonlinear nuclear oscillator.

In Fig. 16, we show the distribution function of the values of determinants of matrices of a transformation, which is calculated by the experimental points generated by the attractor.

For the use in SIF, we choose the mappings with the largest probability, by using the distribution of the modulus of the determinant of a Jacobian of transformations, which is ordered by the frequency of its appearance in the

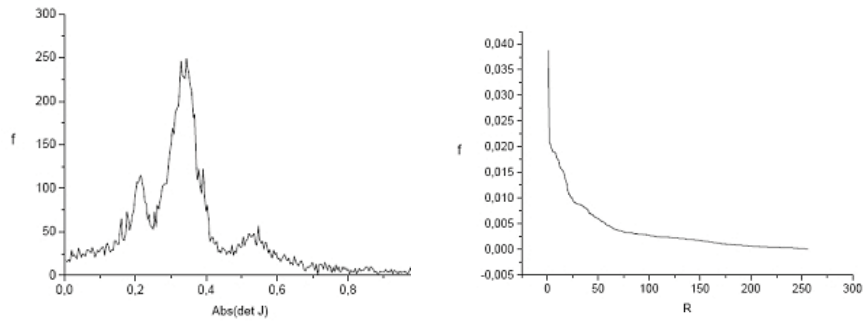


Fig. 16. Distribution function of determinants of matrices of a transformation on the phase plane. On the right is the distribution function of values of a determinant, which are ordered by a decrease of the probability.

dynamics. It is seen from Fig. 16 presenting the distribution of the values of Jacobians that the probability of the values of a Jacobian decreases rapidly.

Such behavior of the distribution allows us to significantly reduce the number of coefficients of the affine functions, which should be determined with the help of an optimization procedure, by using several mappings with the largest frequency of repetition as the basis of SIF. To develop an efficient method of optimization and, hence, of reconstruction of the operators of evolution, it is necessary to choose the functionals for the determination of a proximity of the input attractor and the attractor reconstructed with the help of the regularization procedure.

In other words, we need an efficient procedure of classification of attractors on the basis of the analysis of trajectories in order that the reconstructed trajectory be in the same class of trajectories (attractors), as the initial one.

By the theorem proved in the theory of SIF, a collection of linear operators that transfer the system with a quadratic rate in a required state can be found for any attractor in the input phase space. In other words, the appropriate electromagnetic and entropic control exists for any final state of the nuclear system.

The above-considered description of the evolution of a nuclear system under the action of entropic and electromagnetic fields is true in the general case and, hence, is applicable also for the initially unstable systems, for example, participating in beta-processes. In beta-processes at the constant mass number, the charge number can vary by 1 with a definite probability at a fixed value (attained to the time moment of the decay) of fractal dimension. In this case, for the efficient description of the nuclear process, it is necessary to add the activity or the probability of the beta-process to a series of parameters of the system including the fractal dimension. The half-lives for different attractors characterizing the final states of the nucleus are different, and the state arisen as a result of the evolution is a mixture of nuclei with various probabilities of decay and the concentrations of the corresponding components.

It was shown in the cycle of works [1–3] that the coherent acceleration exceeding the threshold determined by dissipative processes induces the space-

time curvature

$$\kappa(\eta) \approx 2 \frac{a_{cog}^2}{c^4}.$$

Thus, a change of the character of the process of random decay of unstable nuclei, which were independent in the inertial reference system, characterizes, on the one hand, the appearance of correlations in the system of decaying nuclei. On the other hand, it is related to the appearance of the space-time curvature. In other words, a change of the character of the decay of unstable nuclei is determined by the space-time curvature and can be used for the measurement of the curvature.

Correlations in a system are characterized by the nonequilibrium parameter q and the order parameter η . The same parameters determine the exponent in the law of time variation of the activity of isotopes and are directly connected with the coherent acceleration of the system, on which the space-time curvature depends as well. We now find the relations allowing us to estimate the connection between the decay rate of nuclei, order parameter, and curvature.

The connection of the parameters q and η and the coherent acceleration a_{cog} was studied in [1–3] and takes the form

$$q(\eta) = \begin{cases} q_- = 1 - \eta, & q \leq 1 \\ q_+ = \frac{1}{1 - \eta}, & q > 1 \end{cases}, \quad q = \sqrt{1 + \gamma_a \frac{a_{cog}}{|a_{dis}|}}, \quad (4.6)$$

where $|a_{dis}|$ is the acceleration due to the dissipation in the medium. Let us use relation (4.6) to estimate the dependence of the space-time curvature on the order parameter

$$\kappa(\eta) \approx 2 \frac{a_{cog}^2}{c^4} \varepsilon^2 \approx 2 \left(\frac{a_{cog}}{a_{dis}} \right) \left(\frac{u_T^2}{l_{st}} \right)^2 \frac{\varepsilon^2}{c^4} \approx \left(\frac{q(\eta)^2 - 1}{\gamma_a} \right)^2 \frac{1}{l_{st}^2} \beta_T^4 \varepsilon^2, \quad (4.7)$$

where l_{st} is the free path length of particles in the system, ε is the dielectric permittivity of the medium. At small order parameters, the space-time curvature can be approximately estimated from the relation $\kappa(\eta) \approx 4 \frac{1}{l_{st}^2} \beta_T^4 \varepsilon^2 \eta^2$.

Introducing the curvature radius $R_\kappa(\eta)$, we obtain

$$\frac{R_\kappa(\eta)}{l_{st}} \approx \frac{\gamma_a}{\beta_T^2 \varepsilon} \frac{1}{|q(\eta)^2 - 1|} \approx \frac{\gamma_a}{\beta_T^2 \varepsilon} \frac{1}{\eta}. \quad (4.8)$$

In the general case, the correlations in the states of nuclei lead to the power laws of decay. The clear deviation of the processes of decay from the exponential law is traced in all experiments with the action of mass forces on decaying nuclear systems. Determining the exponents in the law of decay by experimental data, we can estimate the curvature radius of the space-time induced by the mass forces of the attain level in the system, by using (4.8).

5 Realization of the processes of collective synthesis and the control of nuclear processes

The theoretical studies presented in [1–3] and in the above sections of this work are the basis for the design and fabrication of the first experimental installations of two types realizing the conclusions of the theory of control of the dynamical systems with varying constraints and give the basis for the development of a new paradigm of nuclear technologies:

- In special electro-dynamical systems of various types (resonators with a dielectric filler and coils with dielectric cores) undergoing the action of self-consistent superwide-band electromagnetic fields, a control of the half-life of radioactive elements in large limits is realized.
- On the installation realizing the initiation of a large number of shell-like structures (bubbles) in various fluids, we register the processes of explosive energy yield that exceeds significantly the energy taken from the supply unit of the installation.

The scenario of realization of the collective nuclear reactions of synthesis is based on the processes of initiation and evolution of shell structures running in the noninertial reference system, which is determined by a coherent acceleration of the system. In the general case, the dynamical systems evolve as multiscale ones, i.e., as the systems possessing all spatial scales from the largest to the smallest one. Below, we indicate the basic elements of the control of nuclear processes that follow from the general theory and are realized in the above-mentioned installations.

The main types of “ignitors” for the implementation of the synthesis and the control of nuclear processes are electromagnetic drivers and macroscopic sources of the negentropy (information) flow, as well as a mechanical impact in some cases (e.g., in the systems with bubbles in a fluid). The contributions of all drivers to a change of the momentum of particles of the system are summed: $|\Delta p| \leq |(\Delta \vec{p})_m| + |(\Delta \vec{p})_A| + |(\Delta \vec{p})_S|$. Here, we account for the following main channels of transfer of the momentum to particles:

- Macroscopic hydrodynamical impact, which induces a shock increment of the momentum of particles (the impact can be made, in particular, by longitudinal acoustic waves in a medium):

$$(\Delta p)_m \approx m \Delta u_{sh}; \quad (5.1)$$

- Direct shock increment of the momentum in an electromagnetic field:

$$(\Delta p)_A \approx qA, \quad (5.2)$$

A is the vector potential of the electromagnetic field;

- Increment of the momentum in the entropy gradient field:

$$(\Delta p)_S \approx \vec{p}_s, \quad (5.3)$$

$\vec{p}_s = \frac{\hbar}{2} \nabla S$ is the entropic pulse.

If the conditions of dominance of the driver-caused perturbations of momenta $|\Delta p|$ over the chaotic thermal momenta of particles p_T are satisfied, a coherent accelerated motion arises in the system. In this case, the arisen coherent acceleration transforms the input inertial system in a noninertial one, and the space-time becomes anisotropic (see [3]).

As is well known, the symmetries of the space-time define the appropriate conservation laws by the Noether theorem. The presence of a symmetry makes the system closed, and the symmetry breaking makes it open along the relevant "channel." Since the action of a mass force induces an anisotropy of the space, the system becomes open along the "channel" of the angular momentum, i.e., the angular momentum in such bound system of particles is not conserved. The action of mass forces on the system of particles is manifested in the appearance of not only an additional entropic momentum \vec{p}_s of the particles located at a point with radius-vector \vec{r} , but the corresponding angular momentum \vec{M}_s ,

$$\vec{M}_s = [\vec{r}, \vec{p}_{s_s}]. \quad (5.4)$$

Thus, the action of mass forces on the system of particles, which is terminated by the formation of a noninertial reference system, leads to the appearance of a rotation that can be interpreted as the Thomas precession [10]. In other words, the orientation of any given vector \vec{s} (e.g., the spins of particles) is changed due to the coherent acceleration and because the system becomes noninertial.

In the nonrelativistic case, we have approximately

$$\frac{d\langle \vec{s} \rangle}{dt} \approx \frac{u}{c} \cdot \frac{a_{cog}}{c} |s|.$$

If a noticeable magnetic field acts on a system of particles, then we should consider the Larmor precession of a spin in the magnetic field. In this case,

$$\frac{d\langle \vec{s} \rangle}{dt} \approx \left(\omega_B + \frac{u}{c} \cdot \frac{a_{cog}}{c} \right) |s|, \quad \omega_B \approx \frac{eB}{mc}. \quad (5.5)$$

The action of a driver of mass forces on the system of particles causes, if the conditions of dominance are satisfied, the separation of a subsystem of particles (shell) with mass number A_{sh} , where the mass force acts, and the coherent part composed of A_{shCog} particles with the initial parameter of order $\eta_{sh} = \frac{A_{shCog}}{A_{sh}}$ is formed.

In the general case, the system, which is isotropic at the initial time moment and possesses the spatial distribution of particles with characteristic scale l_0 , evolves due to the coherent acceleration in a deformed anisotropic state with a large number of external spatial scales. The basic classification of methods and scenarios of the synthesis with its all phases and stages can be performed namely by the method of creation of the germs of anisotropic shell structures.

The dominant perturbation must ensure the anisotropy and the space-time curvature, as minimum, in the spatial region occupied by a germ of shell structures, as well as the order and nonequilibrium parameters (η_{sh}, q_{sh}) related to the curvature. In this case, the quasistationary spatial scales vary:

- $l_- < l_0$ in the direction of localization;
- $l_+ > l_0$ in the orthogonal directions of delocalization.

The processes of structure-forming, which are initiated by the input driver, reveal themselves, in turn, by means of the self-consistent drivers of mass entropic forces. As a result, the entropy and the specific entropy per particle are changed, respectively, by the values

$$(\Delta S)_b \approx 0.038(\eta_{sh} A_{sh})^{3/2}, \quad \frac{(\Delta S)_b}{A_{sh}} \approx 3.8 \cdot 10^{-2} \eta_{sh}^{3/2} A_{sh}^{1/2}. \quad (5.6)$$

If the condition of dominance is satisfied, the action of a driver leads to the coherent acceleration

$$\begin{aligned} a_{cog} &\approx \frac{1}{m_p} \frac{d}{dt} (\Delta p) \approx \frac{1}{m_p} \left(\frac{\alpha_d p_T}{\tau_{eff}} \right) \approx \alpha_d \frac{u_T}{\tau_{eff}}, \\ \alpha_{dS} &\approx 0.38 \left(\frac{\lambda_{D-B}}{R_{sh}} \right) \eta_{sh}^{3/2} A_{sh}^{1/2} \end{aligned} \quad (5.7)$$

and to the evolution of the structure of the system (see the principle of dynamical harmonization [1]).

The appearance of the angular momentum \vec{M}_s of particles is physically related to the following circumstance. The action of the coherent acceleration on a system of particles causes, as is known (see [3]), the appearance of connections between particles of the system through a change of the structure of the space and creates an anisotropy of the physical vacuum and the space on the whole. These processes are defined by that the space-time geometry is directly connected with vacuum structures, and the total momentum of an open system of particles can be written as follows:

$$\vec{M} = \vec{M}_0 + \vec{M}_s \quad (5.8)$$

The additional entropic term \vec{M}_s is related to a rearrangement of the space-time geometry and to the inflow of the additional rotation energy into the system of coupled particles moving under the action of the coherent acceleration and clarifies the physical essence of the Thomas precession [18] arising in noninertial systems.

Definition (5.4) of \vec{M}_s implies directly that the contribution to the angular momentum of particles from entropic forces varies in time in correspondence with the equation:

$$\frac{d\vec{M}_s}{dt} = [\vec{u}, \vec{p}_s] + \frac{\hbar}{2} [\vec{r}, \nabla \dot{S}]. \quad (5.9)$$

If we use the relation between a change of the binding energy ε of the system and the entropy production [3]

$$d\varepsilon = \frac{\hbar}{2} d(\sigma_S), \quad \sigma_S = \dot{S}, \quad (5.10)$$

which generalizes, in fact, the Heisenberg principle of uncertainty for the energy and the time in the systems with varying constraints,

$$\Delta t \Delta \varepsilon \approx \frac{\hbar}{2} \Delta S, \quad (5.11)$$

then Eq. (5.9) can be written in the form

$$\frac{d\vec{M}_s}{dt} = [\vec{v}, \vec{P}_s] + [\vec{r}, \nabla\varepsilon]. \quad (5.12)$$

It is seen from this equation that the open systems with a coherent acceleration are characterized by the direct connection between the appearance of a rotation and the rearrangement of constraints in the system, which are generated by a change of the space-time structure (see [3]).

It is significant that the interaction of the electromagnetic field with a spin system of particles leads to a source in the equation for the entropy [3, 11]. Since all particles of the matter have spin, it turns out that the entropic mass force, which is formed in this case, acts on all material objects irrespective of their charge.

As is well known, E. Cartan developed a theory [12] extending Einstein's general relativity theory. In his theory, the matter characterized by the tensor of energy-momentum and the spin serves as a source of the metric gravitational field and the torsion field. In our case, the system of spins acts on the Riemann space time and forms the Cartan–Einstein space, by setting its torsion. Thus, the external electromagnetic field launches the process of self-organization of the matter with the participation of the Cartan–Einstein space.

It is clear that if all positions of particles in the system are equivalent for the action of a mass force, then the good zero approximation is a nonequilibrium system with spatially and statistically homogeneous fractal properties, where the basic evolution occurs in the energy component of the phase space.

The analysis of the asymptotic behavior of solutions of the kinetic equations for a system of particles at a large impact parameter leads to the distributions over energies with power asymptotic [3]:

$$f(\varepsilon) \approx \frac{1}{1 + \alpha_s \exp_q \left(\frac{\varepsilon - \mu}{T_{eff}} \right)}, \quad q = \sqrt{1 + \alpha_I Q_{imp}}. \quad (5.13)$$

Here, q is the nonequilibrium parameter, and the solutions were written with the use of the quasipower generalizations of exponential functions

$$\exp_q(x) = (1 + (q-1)x)^{1/(q-1)}.$$

The power asymptotics testifies to the appearance of a scaling invariance in the system and the strong correlations of the states of particles.

We can say that the power asymptotics of the distribution functions (and of the wave functions) of the system are determined by the ratio of the intensity of a mass force and the intensity of the dissipation of energy in heat, and the coefficient α_I in (5.13) characterizes the information introduced by the mass force in the dynamical system.

All drivers, including the input and self-consistent ones, contribute to the coherent acceleration, impact parameter Q_{imp} , perturbation of the nonequilibrium parameter q , and, respectively, the order parameter η . We now write the formula for the impact parameter in the form convenient for the estimates:

$$Q_{imp} \approx \frac{\Delta p}{m_p u_T} \frac{\tau_{dis}}{\tau_f} \approx \alpha_d \frac{\tau_{dis}}{\tau_f}. \quad (5.14)$$

In our theory, the particular property of the impact parameter Q_{imp} is its dependence on the characteristic duration of the impact action in a power of minus three. The degree of nonequilibrium and the impact parameter Q_{imp} of states of a physical system determine the order parameter with the help of the relations

$$\eta \approx \frac{q-1}{q} \approx \frac{\sqrt{1+Q_{imp}}-1}{\sqrt{1+Q_{imp}}}, \quad Q_{imp} = \frac{A^2}{\rho T} \frac{\tau_{dis}}{\tau_f^3}. \quad (5.15)$$

In this case, the strong correlations arise due to the coherent acceleration via the curvature of the anisotropic space-time [3]:

$$r_{xp} \approx \eta^{0.553} \approx \left(1 - \frac{1}{\sqrt{1+Q_{imp}}}\right)^{0.553}. \quad (5.16)$$

In the general case, the correlations r_{xp} between momenta and coordinates in the uncertainty relation were considered by H. Robertson and E. Schrödinger [13], who wrote it in the form

$$\Delta x \Delta p_x \geq \frac{\hbar}{2\sqrt{1 - \left(1 - \frac{1}{\sqrt{1+Q_{imp}}}\right)^{1.1}}}. \quad (5.17)$$

The uncertainty relation (5.17) written for two orthogonal directions qualitatively characterizing the shell structures with increased correlations r_{xp} defines the connection of the scales l_+ and l_- :

$$l_+ \geq l_- \frac{1}{2\pi} \sqrt{\frac{3}{8}} \left(\frac{\hbar}{p_f l_-} \left(1 - \left(1 - \frac{1}{\sqrt{1+Q_{imp}}}\right)^{1.1}\right)^{-1/2} \right)^{3/2}. \quad (5.18)$$

The explosive growth of the scale of long-range correlations l_+ in the system, which is accounted in relation (5.18), allows one to control the permeability of Coulomb barriers, rates of nuclear processes, and synthesis of isomers, as is shown in the present work.

5.1 Control of unstable nuclear systems

For the practical application of the principle of dynamical harmonization, it is necessary to be able to initiate the adequate types of mass forces in the system. It is obvious that it is easier to affect the processes in an unstable system and to control it than in a stable one. Therefore, the problem of control of the activity of unstable isotopes was solved historically in the first turn.

5.1.1 Statistical properties of experimental series of the decay activities for isotopes

To realize the control of the lifetime of unstable isotopes and the synthesis of fractal isomers, we developed the special electrodynamical systems of various types, in which the initial monochromatic oscillations were transformed

into superwide-band electromagnetic oscillations with the controlled width of a spectrum due to the use of resonators and coils filled in with dielectric materials with a nanostructure.

The created electromagnetic fields were used as the input drivers of mass forces for isotopes. The basic experiments were carried out on specimens of sand and granite with a sufficient initial activity due to inclusions of the isotopes of cesium, strontium, and radium. The schematic diagram of the installation for the treatment of isotopes is given in Fig. 17.

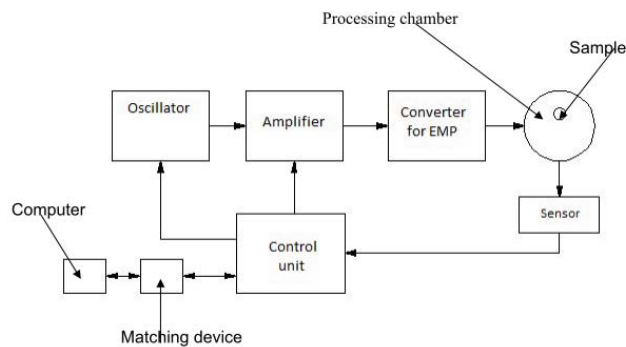


Fig. 17. The schematic diagram of the installation for the treatment of isotopes.

A change of the half-lives of isotopes under the action of electromagnetic drivers is well demonstrated by the analysis of experimental series of the activities of decays of ^{90}Sr in two cases: for the free decay of radioactive nuclei and for the decay of nuclei after the action of entropic and electromagnetic fields. In Figs. 18 and 19, we show the time series of the decay rates for free nuclei and nuclei after the treatment with entropic and electromagnetic fields.

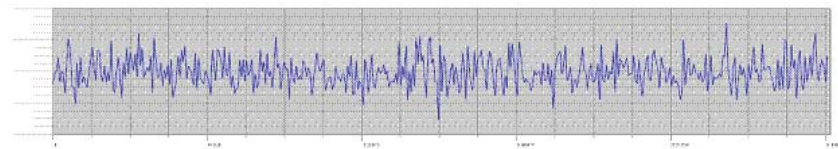


Fig. 18. Time series of the decay rates without action of electromagnetic fields during one hour.

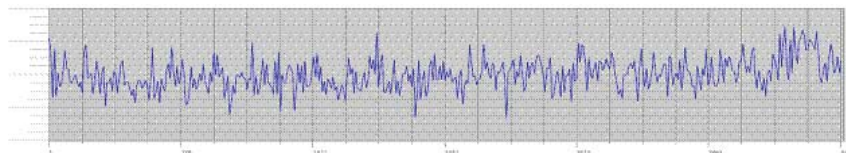


Fig. 19. Time series of the decay rates after the action of electromagnetic fields during one hour.

The autocorrelation functions for these two cases are presented in Figs. 20 and 21, respectively. The action of the electromagnetic field is clearly revealed in an increase of correlations in the system.

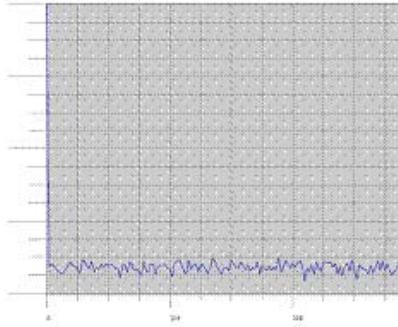


Fig. 20. Autocorrelation function of a series of the decay rates without action of electromagnetic fields (realization is given in Fig. 18).

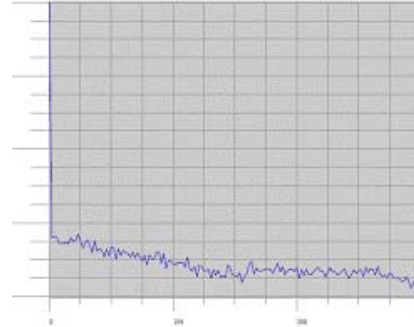


Fig. 21. Autocorrelation function of a time series of the decay rates under the action of electromagnetic fields (realization is given in Fig. 19).

Comparing Figs. 20 and 21, we can see the start of the formation of a structure of attractors in a nuclear dynamical system under the action of electromagnetic fields. The further studies will clarify the dependence of the deformations of attractors on the parameters of controlling electromagnetic fields.

The long-term measurements of the activities of isotopes with a large half-life during one year and more allow us to analyze the large-scale variations of the activity of isotopes and their specific features.

The theory yields the regular essential increase of correlations in a nuclear dynamical system under the action of coherent accelerations. The analysis of experimental results demonstrates the appearance of correlations and attractors. The reconstructions of the phase space of a dynamical system by the series of activities of an isotope in the free state and under the action of the electromagnetic field are shown in Figs. 22 and Fig. 23, respectively.

The existence of a correlation of the states of decaying nuclei leads to the law of decay that generalizes the exponential one and is the exact solution of the equation for the process of decay with regularized operators, in particular, with fractional operators possessing the indices, whose values are determined by the attained values of correlations, the order parameter η , and the nonequilibrium parameter q for nuclei after the action of mass forces on them, which induces the coherent acceleration a_{cog} .

5.1.2 Comparison of theoretical and experimental data on the activity of a mixture of isomers

The results obtained under the action on a Sr isotope are the most demonstrative. The experiments were carried out on three specimens. The control specimen of Sr showed the activity during one year, which corresponded exactly to the exponential law of decay $A_t = \frac{0.693}{\tau_0} N_0 \exp\left(-0.693 \frac{t}{\tau_0}\right)$ with the half-life $\tau_0 \approx 28.8$ yr and the number of active nuclei $N_0 = A_t \frac{\tau_0}{0.693} \approx 1.1710^{12}$ at an activity of about 900 decays/min.

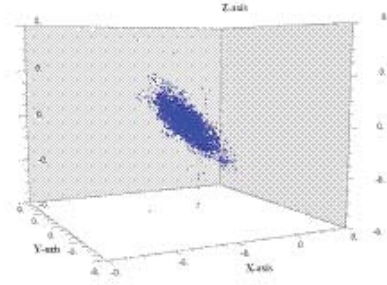


Fig. 22. Phase space of a dynamical system in the free state.

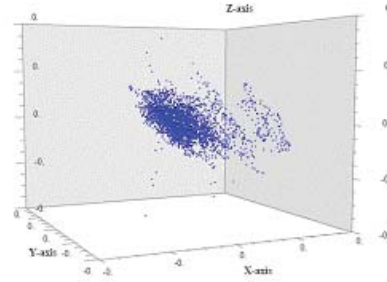


Fig. 23. Phase space of a dynamical system after the action of an electromagnetic field. The exponential decay of nuclear systems is related to the Poisson statistics of elementary events. However, this statistics is valid for independent events. In the presence of strong correlations, it should be replaced by a statistics taking the memory of the system into account.

The intense action of high-frequency electromagnetic fields with longitudinal polarization on a specimen in a chamber with dielectric inserts for three days initiated the processes of formation of fractal isomers of Sr and a drop of the activity observed for one year (see Fig. 24).

On the basis of these data, we obtain the estimates of a composition of isotopes after the treatment, which are given in Section 3.3. To correct the law of decrease of the activity, the data of measurements of the activities (see the relative values of activities in Fig. 24) are shown in Fig. 25 on the double logarithmic scale.

It is seen from Fig. 25 that the dependence of the activity logarithm on the time logarithm for one year can be well approximated by a linear function, which shows obviously the power dependence of the activity on the time in correspondence with the asymptotics following from our theory.

We have performed the experiments with pulse and wide-band sources. In Figs. 24 and 25, we indicate the regions, where the corresponding sources start to act. The presented data allow us to assume that the action with pulse sources causes, as is seen from the plots, a growth of the number of isomers with the mean half-lives of about a half-year. The start of a treatment involving wide-band sources led to an increase of the contribution of rapidly decaying isomers. The last section of the plot in Fig. 19 indicates the formation of isomers with the half-life less than that after the first action and also with the power law of decay of isomers.

We also obtained the interesting data on the influence of electromagnetic drivers on a Cs isotope. In the initial state without any action, the series of activities have practically no correlations, and the phase trajectories are a single chaotic attractor (see Fig. 26).

As a result of the action, the initial nucleus transits in a state with two attractors. The mass numbers of components are approximately equal to one another. Since the processes realized in the described dynamics are the pro-

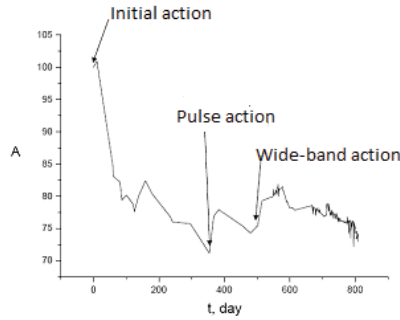


Fig. 24. Measured dependence of the activity of Sr on the time. Initial action. Wide-band action. Pulse action

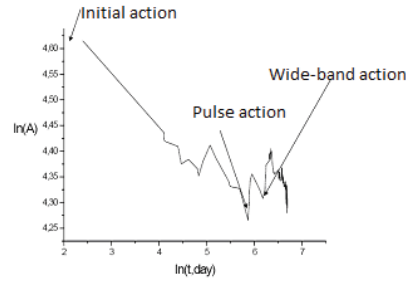


Fig. 25. Logarithm of the activity of a radioactive substance after the treatment versus the logarithm of the time. The linear dependence corresponds to the power dependence of the activity on the time.

cesses of self-organization, the evolution occurs to the side of the formation of maximally stable structures for the least time.

In Fig. 27 presenting the experimental data, the growth of the activity occurs in the periods, where the radiation source starts to operate. In sufficiently long time intervals without radiation, we observe a drop of the activity. Such behavior of the activity is related to the transformation of the initial isotope in quasistable and short-lived isomers during the action of radiation.

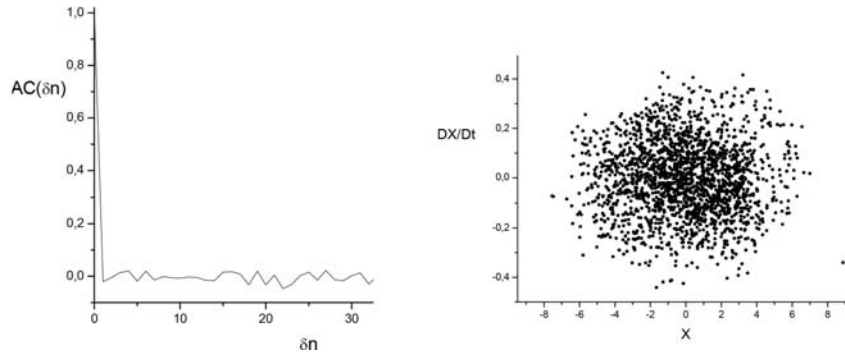


Fig. 26. Autocorrelations and the phase portrait obtained for a time series of activities.

When the radiation stops to act, the production of isomers due to the action of the external source ceases as well. The short-lived isomers transit in a stable state, and a growth of the activity is related to the creation of stable isomers due to the decay of quasistable states. Such behavior is confirmed by the time dependence of the entropy (see Fig. 28).

In the time intervals where a source acts, the growth of structures exceeds their decay, and the entropy decreases. If the source is switched-off, the structures decay, and the entropy increases sharply.

During the action of an electromagnetic field in the radiofrequency range on isotopes, the spectrum widens as a result of the interaction with a dielectric, the radiation is saturated by the components of electromagnetic fields with longitudinal polarization, and the mass forces are formed.

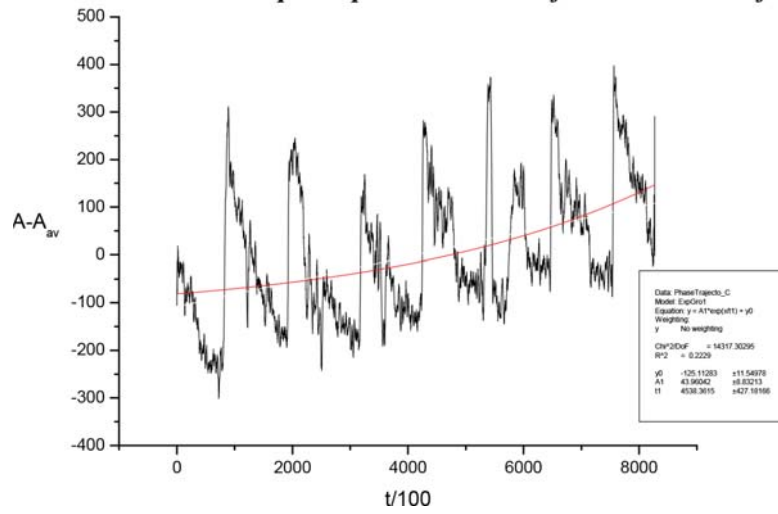


Fig. 27. Dynamics of the activity of Cs^{137} under the action of electromagnetic pulses.

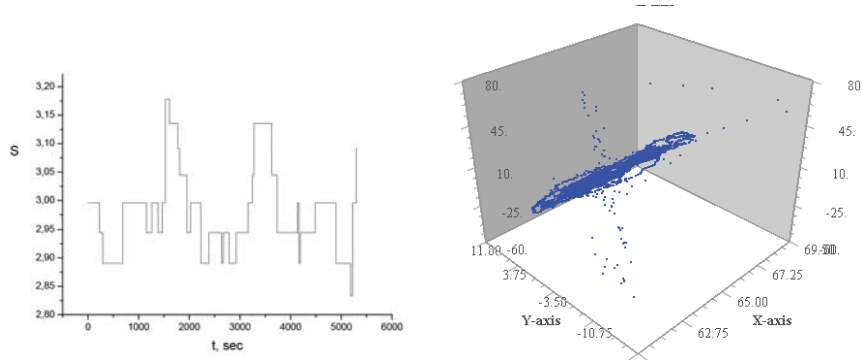


Fig. 28. Time dependence of the entropy at the action of a wide-band radiation (left) and phase trajectories at a low-energy wide-band action (right).

These processes result in the increase of correlations in the decay processes, which testifies to the growth of structures in nuclei. The processes of growth of structures are manifested in the dynamics of the nuclear system, i.e., in the structure of its phase space (see Fig. 29). A structure of the phase space of a nucleus begins to reveal itself quite clearly after such action.

One of the components of nuclei of a mixture of isomers, which is formed under the action of mass forces, turns out at once in the region with the maximal stability, i.e., it is a fractal isomer of a nucleus from the iron-nickel peak. The second nuclear system is also a fractal isomer of a nucleus-residual. All isotopes supplementing the stable component to the parent nucleus have practically always a small lifetime of about seconds. The formation of a fractal isomer increases their stability by orders, by enhancing the half-life to several hours. The described pattern corresponds to the experimental data obtained with the use of wide-band fields (see Fig. 27).

The initial activity of the isotope $A_0 = 65.4$ decays/sec, and the mean activity for ten days $A_{av} = 66.65$ decays/sec. The intensity is significantly increased after each treatment and then decreased, but not to the initial level.

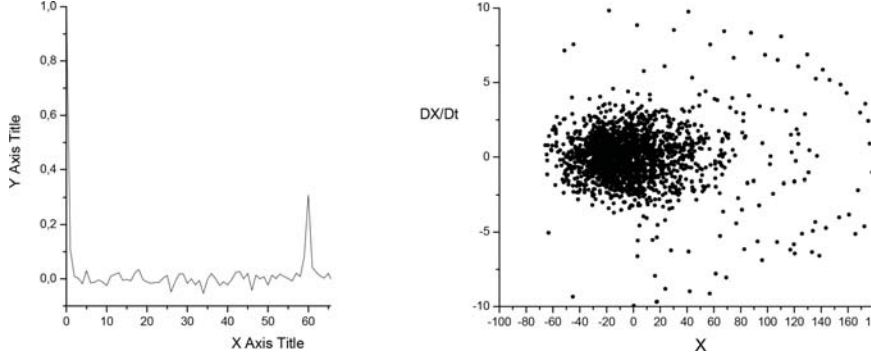


Fig. 29. Autocorrelation function of a series of activities (left) and the structure of the phase space (right) under the radiofrequency action.

As distinct from the equilibrium situation where the activity drops by the law (t is measured in seconds)

$$A(t) = 65.4 \exp\left(-0.693 \frac{t}{\tau_{1/2}}\right), \quad \tau_{1/2} = 9.51 \cdot 10^8, \quad (5.19)$$

the mean activity on the initial stage of the process after the action of pulses of an electromagnetic field increases by the law

$$A_{av}(t) = 66.65 + 0.439 \exp\left(\frac{t}{\tau_s}\right), \quad \tau_s = 4.538 \cdot 10^5. \quad (5.20)$$

The increase of the activity is related to the transformation of input isotopes in isomers, i.e., the activity of isomers behaves itself on the average as $A_{isomer} = 0.439 \exp\left(\frac{t}{\tau_s}\right)$. This activity corresponds to the exponentially increasing number of isomers

$$N_{isomer} = 0.63 \exp\left(\frac{t}{\tau_s}\right) \tau_s. \quad (5.21)$$

On the whole, the analysis of the obtained experimental data on Cs and Sr indicates the close half-lives of arisen isomers of the order of $\tau_{is} \approx 10^4$ – 10^5 sec and the characteristic times of the growth of the number of isomers in the same interval. It is clear that the growth of the activity on the initial stage of the evolution of isomers is replaced by a decrease that reveals itself on great times.

By integrating the activity of the isotope and by calculating the number of decayed nuclei for the time interval, when the mean activity increases (see Fig. 27), we obtain that the number of really decayed nuclei is by 17% more than that without any action. The comparison of experimental and theoretical curves of the decay is given in Fig. 30.

The curve in Fig. 30 (b) approximates the power dependence of a decrease of the activity. The exponent of the decrease is determined by the ratio of the coherent and dissipative accelerations and corresponds to the order parameter $\eta \approx 0.1$.

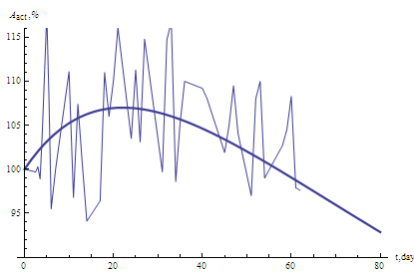


Fig. 30 (a). Experimental and theoretical dependences of the activity of Cs on the time.

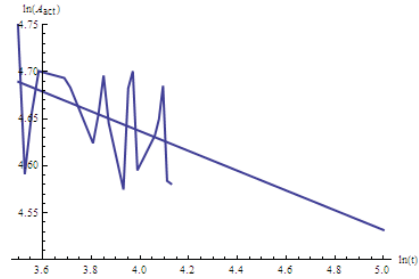


Fig. 30 (b). Logarithm of the activity of an isotope versus the logarithm of the time.

After the treatment of radium in the composition of granites, we observe the tendency to a decrease of the activity with the indicated characteristic half-life of the order of 10^7 sec. In this case, the activity drops by 15 % for the time interval of about two months.

This corresponds to that about 90 % of Ra nuclei become stable fractal isomers, and the small share of fractal isomers (of the order of 0.1 %) with a half-life of 10^7 sec ensures the activity measured after the experiment.

We now present the main qualitative conclusions of the analysis of the time series of activities:

1. Power dependence of activities on the time indicates the appearance of new structures in nuclei and the existence of strong correlations.
2. Evolution of the entropy along different directions for different phases of the experiment confirms the informational character of the action.
3. Inhomogeneity of the phase space of a dynamical system related to decays confirms the presence of many components in a mixture of isomers arising under the action of external drivers.
4. Existence of sections with a growth of activities indicates that the used drivers create a high coherent acceleration and, respectively, increase the order parameter of a mixture of isomers from 0 to 0.2 – 0.3. Namely in this case, we have the dimension $D_f \approx 2.6$, which ensures the realization of induced decays with the observed dynamics.

5.2 On a promising technology of nuclear fusion in coherent multiscale systems

The results obtained in [1–3] can be applied not only to the design of a technology of control of the evolution of unstable nuclei (see the previous section), but can serve as the basis of the creation of new environmentally safe and powerful sources of the nuclear energy, by presenting the substantial answers to two basic questions:

- How can the problem of the increasing world needs in energy be solved in the most harmonic and environmentally safe way?
- Which key phenomena and effects can serve as the basis for the development of a new efficient technology of production of the energy with the control of energy potentialities in the Nature surrounding us?

It is obvious that all primary sources of energy in the Nature have a single basis: the processes running with a change of the binding energy of systems. This concerns the most spread sources of energy based on a transformation of the binding energy on the atomic and molecular levels, as well as the nuclear processes generated by a change of the binding energy of nucleons in the nuclei of atoms. The most powerful sources of energy are those used the binding energy of namely many-nucleon nuclear systems, since the density of this energy is (according to the available ideas) of the order of several MeV per nucleon as distinct from the chemical energy which is several eV per atom or molecule.

All ways to the solution of the energy problem, which are widely discussed now, cannot yet be considered quite satisfactory (even at their efficient development). Indeed, they are not sufficient by the amount of the produced energy or their realization leads to the creation of new hardly solvable problems, including the safety problems and the ecological problems related to the accumulation of radioactive waste products.

The answers to the above-posed questions are formed on the basis of the mechanisms of self-organization and the factors controlling the evolution of multiconnected dynamical systems of any physical nature, which are studied by the developed theory of dynamical harmonization [1–3]. This theory becomes rapidly the foundation and a working tool for the development of new technologies of control of the nuclear synthesis on the present stage of studies aimed at the production of nuclear isomers as the accumulators of energy and at the fabrication of powerful environmentally safe sources of nuclear energy.

The physical essence of connections between particles (irrespective of the nature of these particles) is determined by their ability to create collective states with appropriate self-consistent values of binding energy and, as a consequence, to change the effective mass of particles in transient processes (and at phase transitions).

The development of new nuclear technologies of self-organizing synthesis is based on the general ideas developed in [1–3] of the structure of systems and on the comprehension of that the dynamical systems of any nature are not the fixed objects for the application of various forces, but, in some meaning, the optimal structures evolving in correspondence with the principle of dynamical harmonization. This principle defines the targeted “behavior” related to the development of their internal structure along the direction of maximal stability and minimal dissipation of energy in the transient processes induced by the action of external forces.

It was shown in [2] that the variations of a fractal internal structure of nuclei allow one to extend the possible range of their specific binding energies by several times and to significantly change the conditions of stability of nuclear systems (see Figs. 31 and 32 taken from [2]). The results obtained yield, in particular, the existence of

- superstable nuclei in the region of mass numbers of nuclei from the Mendeleev Periodic table (e.g., a stable fractal nuclear isomer with $Z = 28$, $A = 70$, $D_f \approx 2.45$, binding energy of 15 MeV per nucleon, and stability reserve of about 4 MeV per nucleon) and superheavy nuclei with high specific binding energy and large stability reserve, i.e., nuclei with $A < 60000$, which exceed all known nuclei from from the Mendeleev

Periodic table by mass numbers (e.g., a superheavy nucleus with the following parameters: $A = 39000$, $Z = 5770$, $D_f \approx 2.62$, specific binding energy of 18.49 MeV per nucleon, and stability reserve > 4 MeV per nucleon);

- superheavy stable nuclei with high specific binding energies, but with a small stability reserve (e.g., a superheavy fractal nuclear isomer with the mass number $A = 19000$ and the charge number $Z = 1580 - \frac{19000}{1580} A_{D_f=2.69}$, which have a stability reserve from tens to hundreds of keV per nucleon at small variations of the fractal dimension and can participate in induced decays [2]);
- nuclei with low binding energies and a small stability reserve (e.g., a fractal nuclear isomer with $Z = 40$, $A = 90$, $D_f \approx 2.76$, deformation $\delta \approx 0.17$, and the binding energy of about 1 MeV per nucleon, which is separated from the state with higher specific binding energy by the energy barrier of at most 0.5 MeV per nucleon).

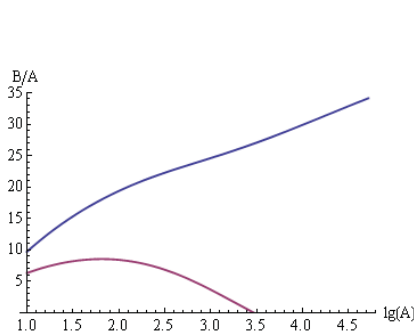


Fig. 31. Specific binding energy of fractal nuclear structures on the drip line. The lower curve corresponds to ordinary nuclei in the form of liquid drops. The upper curve corresponds to nuclei with a fractal structure.

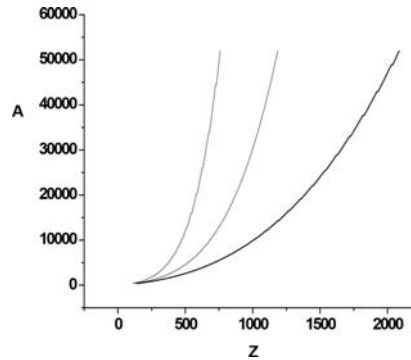


Fig. 32. For each mass number, there exists the optimal value of charge number, which corresponds to optimal structures with maximal binding energy (the middle curve corresponds to the specific binding energy on the drip line. The values of binding energy are shown in Fig. 31). A deviation from the optimal charge number to the side of its decrease or increase decreases the binding energy. Two other curves correspond to the zero binding energy.

It was shown on the basis of experimental data in the previous section that the possibility to evolve to stable or unstable fractal isomers allows one to vary the life time of unstable isotopes in very wide limits. It was noted in works [2–3] that these theoretical results open a possibility to develop basically new nuclear technologies.

The “shell” model of self-organization developed in [1–3] with the use of the ideas of macroscopic quantum objects (MQOs) and mass entropic forces allows us to find a number of practically important means of creation of MQOs and to control their properties, which opens the possibility to initiate the collective self-organizing nuclear synthesis. In this case, the ratio of exo- and endoenergetic processes of the synthesis can be varied in wide limits. In other words, the energy directedness of the synthesis can be controlled.

The general theory [1–3] implies that the main tool of the control of the evolution of nuclear systems and the collective nuclear synthesis is mass forces sufficient for the organization of such coherent acceleration of these systems that initiates the process of creation and the evolution of shell structures with varying constraints.

The collective self-organizing synthesis passes several phases of its development, by starting from the stage of the initiation of shell structures. In the general case, the system, which is isotropic at the initial time moment and has the equilibrium distribution of particles in the space with characteristic scale l_0 , evolves under the action of mass forces (at the excess of a threshold determined by the dissipation in the given system) in a deformed state characterized by a larger number of external spatial scales.

The criteria for the parameters of primary drivers (ignitors), at which the initial states of shell structures with optimal spatial scales and masses are initiated, are taken in dependence on the type of a working medium (also considered as a dynamical system of particles) in use. For example, they are

- parameters of a diode and a target for the collective nuclear synthesis described in [4] in detail;
- parameters of a fluid and the ultrasound action at the initiation of the collective synthesis in a fluid considered in what follows.

The collapse of bubbles in fluids attracts the attention of researchers for a long time as a system convenient for a realization of the nuclear synthesis. Such system is especially attractive for the application of the theory of collective synthesis, since the main elements of a self-organizing system, namely the shell structures, are seen in a system of bubbles in fluids quite clearly.

As a mechanism of formation of bubbles, one considers the breaking of the continuity of a medium (working fluid) under the action of an ultrasound wave (see, e.g., [14,15]). The initial density of a substance in the shell of a bubble with thickness δ_{sh} , which is created in the hydrodynamical process of its birth, exceeds the equilibrium density of the medium by at most 3 times, and the shell thickness (thickness of a double layer) turns out to be $\frac{\delta_{sh}}{R_{sh}} \approx 0.1$, where R_{sh} is the bubble radius. The first phase ends by the formation of bubbles, i.e., structures of the shell type.

In this phase, the drivers must create a dominating perturbation for the evolution of shells, i.e., to transfer an impact momentum to particles that must exceed the thermal momentum p_T defining the dissipation in the system. The conditions of dominance for the drivers of various types were analyzed in [3] and in Sections 2 and 3 of the present work.

In the experimental installations for the collective synthesis in liquids, we use the ultrasound field with frequencies in the MHz range. In this case, the radii of bubbles in water turn out to be about $5 \cdot 10^{-5}$ cm on the average, which corresponds to the theoretical estimates. It is worth noting that the bubble shell contains about 10^{12} nucleons.

The maximal density of bubbles that can be attained in this case $n_{bub\ max} \leq 10^{12} \text{cm}^{-3}$, whereas the real density in our experiments was at most $n_{bub\ max} \approx 10^7 \text{cm}^{-3}$. The total number of nucleons in all shell structures $A_{bub} \approx 10^{19}$ in 1cm^3 . Respectively, the value of order parameter on the macroscopic level is $\eta_{macro} \approx \frac{A_{bub}}{A_{total}} \approx 10^{-4}$.

After the formation of shell structures in the state with a coherent acceleration, the collapse of shell structures starts in the explosive mode, which is accompanied by the increase of the coherent acceleration and the shell density. The theoretical analysis gives that the collapse duration is about 10^{-10} sec. In Fig. 33, we show a typical theoretical dependence of the acceleration along the radius on the collapse duration.

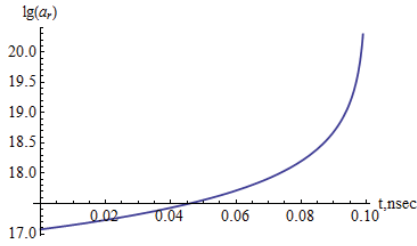


Fig. 33. Typical dependence of the acceleration along a radius (cm/sec^2) on the collapse duration.

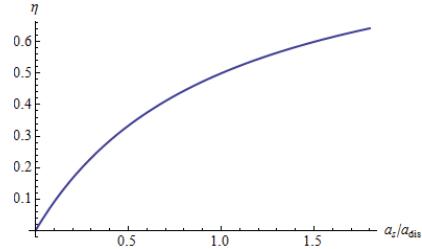


Fig. 34. Dependence of the order parameter on the ratio of the acceleration of the absorbed power to the dissipation acceleration.

We note that the initial acceleration of the dissipation in water at the ordinary temperatures $a_{dis} > 10^{17}$ cm/sec^2 , and it increases in time. For the presented parameters of the ultrasound action, it turns out that the process of collapse of bubbles occurs under conditions of a dominating perturbation along the radius from the very beginning of the process (acceleration of the bubble shell surface is greater than the dissipation acceleration).

The increase of the coherent acceleration causes a growth of the order parameter by relations (5.14) and (5.15). The plot of the dependence $\eta(a_r/a_{dis})$ is given in Fig. 34.

In the general case, the interaction of charged particles in shell structures is realized through the plasma modes of oscillations of a thin plasma layer [1, 4], which propagate in the layer along its surface. The appearance of a positive feedback in the growth of a density and a decrease of the characteristic scale of a structure in a thin layer of the quasineutral medium was first shown by A. Vlasov [16]: “the problem of the existence of a cluster of particles held by the intrinsic forces is related to the existence of the interaction between ions in a partially ionized plasma by means of the exchange by phonons The effect of attraction between ions turns out quite significant. The binding energy of ions attains values of the order of the binding energy of atoms in the solid phase. The binding energy is released at a decrease of the radius of the structure and turns out sufficient to support the conditions for the processes of ionization. The mechanism of the processes of ionization consists in the formation of own inhomogeneous electrostatic fields, which are a consequence of the oscillatory change in the space of the interaction potential of ions through an intermediate system”.

The relativistic effects in a quasineutral medium and the specific electron collapse accompanied by a growth of the density and the ionization degree were considered in works [4, 17–18] in detail. The self-consistent collective electromagnetic and entropic fields in shell structures [1, 3] are namely the intermediate system by A. Vlasov.

The self-consistent ionization of a substance due to the stability loss caused by the action of mass forces occurs, as was mentioned above, in the explosive manner and is accompanied by a change of the number of connections in a shell system. The explosive change of the entropy in time and in space leads to the presence of accelerations of all orders. The mass force arising in these processes in a self-consistent way causes the explosive “flattening” of the wave functions of nuclei.

Thus, the self-consistent drivers of collective nuclear reactions are the self-organizing processes running in the shells of bubbles with the appearance of entropy gradients at the evolving interfaces of bubbles and, hence, with the appearance of coherent entropic accelerations, which can be estimated from the relation

$$a_{cog} \approx \frac{d}{dt} \left(3.8 \cdot 10^{-2} \eta_{sh}^{3/2} A_{bub}^{1/2} \frac{1}{d_{sh}} \right) \approx 3.8 \cdot 10^{-2} \eta_{sh}^{3/2} A_{sh}^{1/2} \frac{1}{d_{sh}^2} \frac{d}{dt} (d_{sh}). \quad (5.23)$$

The scale of influence of the entropy on the processes running in a nucleus can be estimated in view of the fact that the entropy gradients create an additional quantum potential $U_{Ns} = \frac{\hbar^2}{2m} \sum_{i=1}^N \nabla_i^2 S$ in MQOs. This potential decreases the Coulomb barriers (see [3]). This implies that, in order to decrease the Coulomb barrier in a nucleus by U_{Ns} , it is necessary that the entropy be changed at the bubble collapse by the value

$$\Delta S = \frac{2mR^2 U_{Ns}}{\hbar^2 \eta_{sh} A_{sh0}}, \quad (5.24)$$

where R is the bubble radius, η is the order parameter, and A_{sh0} is the number of nucleons in the bubble shell.

It follows from (5.24) that, in order to decrease the Coulomb potential by 1 MeV, it is sufficient to decrease the entropy by $\Delta S = 8.8 \cdot 10^8$. Just such change of the entropy corresponding to the order parameter of a shell $\eta_{sh} \approx 10^{-4}$ occurs at the formation of clusters from $A_{cogsh} \approx 10^7$ nucleons in the shells of bubbles [3].

The increase of the characteristic spatial scales of coherence of the system and the wave functions of nuclei entering the shell structures of bubbles with the order parameter is shown in Figs. 35 and 36 (see also relation (5.18)).

The theories developed by us allow one to consider the nature and the physical mechanisms of bubble nuclear synthesis on a new level.

It might seem that the bubbles, which are a good physical image of MQOs of the “shell” type, are an optimal object for the creation of MQOs and the initiation of the collective synthesis. However, the bubbles involve naturally the mechanism of fracture of MQOs, which would originate in a bubble: the gas with high pressure and temperature, which is compressed by the collapsing bubble, destroys the bubble shell prior to the origination of MQOs in it. Therefore, the collapse of bubbles a fluid without some additional conditions is not the optimal tool for the realization of the process of nuclear synthesis. The suitable objects for the processes of nuclear self-organization are the small-size bubbles, in which the Schwinger mechanism of sonoluminescence is realized.

The basis of this approach is formed by the processes running in the physical vacuum and the controlled change of the correlative structure of its

fluctuations under the action of mass forces of various nature and coherent accelerations induced by them. In this connection, Schwinger's works on the sonoluminescence deserve a particular attention. This luminescence induced by the collapse of bubbles is called the Casimir light [19] and is directly related to the Casimir dynamical effect [20], i.e., to the dynamics of vacuum fluctuations.

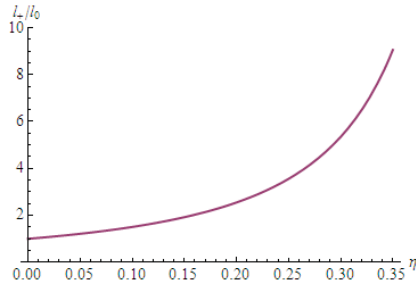


Fig. 35. "Flattening" of the wave function of the system with increase in the order parameter relative to the equilibrium radius.

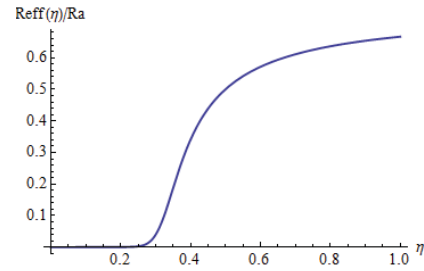


Fig. 36. Effective transverse size of the wave function of a nucleus in units of the mean distance between nuclei versus the order parameter.

The accelerated motion of the dielectric shell of a bubble accompanied by an increase of the order parameter of the system (see Fig. 33, 34) leads to the excitation of fluctuations of the electromagnetic vacuum and the appearance of quanta of light. In the general case, the excited fluctuations of vacuum are nonequilibrium due to the interaction with a substance in the curved space-time and, like a substance, have the nonzero parameter of order η and the corresponding nonequilibrium parameter q different from 1. The mass force of the collapse of a bubble and the appropriate rearrangement of the vacuum structure in it are interconnected, which leads to the luminescence with a wide-band spectrum.

The action of a coherent acceleration leads not only to the appearance of connections in the shell of a bubble and the formation of fractal structures in it, but also to the experimentally observed rotation of the bubble. The nature of this rotation is related to the appearance of an anisotropy of the space under the action of the mass force and, hence, to the nonconservation of the angular momentum of the system (as was shown at the beginning of Section 5, this rotation clarifies the physical essence of the Thomas precession arising in noninertial systems).

It is worth noting that the sonoluminescence is a phenomenon, at which the energy introduced in a medium on the macroscopic level activates the processes on the quantum level. Here, we note that this activation can concern not only the emission of light, but also the rearrangement of a system of connections on the nuclear level of the matter in the shell of a bubble.

The order parameter and the fractal dimension of shell structures (and the nonequilibrium parameter of fluctuations of the physical vacuum) evolve in correspondence with the equation of dynamical harmonization, which can be presented as the Lagrange equations for a change of the structure of a system (see [1], [3], and Section 2). The analysis of the evolution of the system indicates the existence of two basically different versions (see Figs. 35–36):

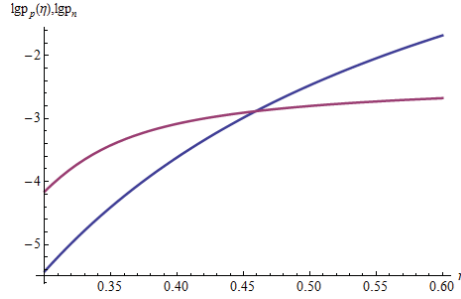


Fig. 37. Probabilities of the reactions of synthesis of nuclear isomers $p_p(\eta)$ (lower curve) and of neutron transfer $p_n(\eta)$ (upper curve) versus the order parameter.

- initial conditions and drivers used for the initiation of collective processes (including the drivers arising in the process of self-organization of shell structures) lead to the bounded relatively small values of order parameter $\eta \ll 0.3$;
- initial conditions and drivers used for the initiation of collective processes (including the drivers arising in the process of self-organization of shell structures) lead to the bounded sufficiently large order parameters $\eta \geq 0.35$.

Let us consider the first case. MQOs do not arise on the scales of shells on the whole. Nevertheless, we have already discussed the formation of clusters in shell structures created by external drivers. The Coulomb barriers in nuclei of clusters are significantly decreased due to the gradients of entropic fields [1, 3]. As a result, the nuclear processes can run even at the indicated small values of order parameter with a sufficiently large probability.

Among these nuclear processes, we indicate two main types:

- nuclear processes of synthesis of fractal nuclear isomers within the clusters of shell structures;
- reactions of neutron transfer at the interaction of the nuclei of some isotopes. These reactions run by virtue of that the natural isotopic compositions of many admixtures (such as sulfur, nickel, titanium, vanadium, mercury, etc.) used by us in the formation of optimal working media contain a large number of combinations of interacting isotopes with the identical and different charges of nuclei, for which the transfer of neutrons is a process with energy gain.

The probabilities of these processes are compared in Fig. 37.

At the values of order parameter $\eta \ll 0.3$, the above-indicated processes ensure the yield of free energy $\Delta w_{MeV}(\eta)$ from 0.1 MeV to several MeV per each of the nucleons entering the coherent subsystems of nuclei, by depending on the composition of a working fluid and the order parameter attained in bubbles η_{sh} ($\eta_{sh} \approx \frac{A_{cogsh}}{A_{totalsh}}$). The initial composition of the medium and the parameter η_{sh} together determine the distribution of products of the synthesis over mass numbers and parameters of the internal structure (fractal dimensions and deformations), i.e., they determine eventually the integral mass defect over the whole spectrum of these products.

The estimate of the energy yield ΔW from a working volume Ω_V cm³ of the fluid per second (i.e., the power of the initiated collective process) can be obtained from the above-given quantities:

$$\Delta W \approx A_{bub}^{3/2} \eta_{sh}^{3/2} p_p(\eta) \Delta w_{MeV}(\eta) \Omega_V \approx 0.6 \eta_{sh}^{3/2} p_p(\eta_{sh}) \Delta w(\eta_{sh}) \Omega_V \text{ Watt.}$$

The experiments aimed at the study of the self-organizing nuclear synthesis were performed on ultrasound cavitation installations, whose photos are presented in Fig. 32. For the typical value $\Omega_V \approx 100$ cm³ (characteristic of all experiments), we obtain the theoretical estimate of the output power at the attained order parameter $\eta_{sh} \approx 10^{-4} \div 5 \cdot 10^{-3}$ in the interval 0.5 kW – 1 kW. Moreover, the yield increases sharply with η_{sh} .

According to the developed model of the process, the effective size of nuclei approaches the mean distance between nuclei, and MQOs are formed (see Fig. 36), as the coherent acceleration attains the level, where the inequality $\eta_{sh} > 0.35$ holds. In this case, the interface of a bubble forms a fractal cluster structure from monomers of the fluid, and the shell is a structure with a fractal distribution of the density of charges. It is a fractal “virtual capacitor”, in which the Coulomb barriers are decreased by many orders due to the gradients of entropic fields [1, 3], and their overcoming is already easy for the nuclei of a coherent multiparticle system. The Coulomb barriers become almost everywhere completely transparent, which creates the conditions for the explosive growth of macroscopic nuclear clusters in the highly coherent layers of the shell with the order parameter close to 1.

The collapse of shell structures representing MQOs is completed by the stability loss for the shells of bubbles. The degenerating shells losing their stability decay into fragments with energies determined by the evolution of the order parameter of a shell at its collapse. By interacting with the surrounding initial substance, the fragments of a destroyed shell form a distribution of fractal nuclear isomers over mass numbers.

The binding energy of fractal isomers is studied in [2] and is presented in Figs. 25 and 26, and the deformations of isomers and the distribution functions were analyzed in the previous sections.

The analysis of the relations obtained in works of this cycle indicates the real possibility to develop, on the modern level, the technique of drivers with parameters ensuring the implementation of multiparticle and multilevel reactions of nuclear synthesis with the free energy yield of about several MeV per nucleon for all amount of created bubbles, including the scenarios based on collapsing bubbles in a fluid.

Using the phenomena of cavitation and sonoluminescence as a working tool of the technology of synthesis, we chose purposefully the media as working fluids and external actions in a series of experiments, whose joint use allowed us to realize a multiscale structure of MQOs from the scales of the working volume of a reaction chamber via the scales of clusters in the shell of a bubble to the nuclear scales.

The self-similarity is the invariance of a system relative to a change of the scale (scaling) is, as was shown in [1–3], one of the most important conditions for the realization of the processes of control. In a sufficiently general case, the system can be represented on the macroscopic level by a dynamical system of

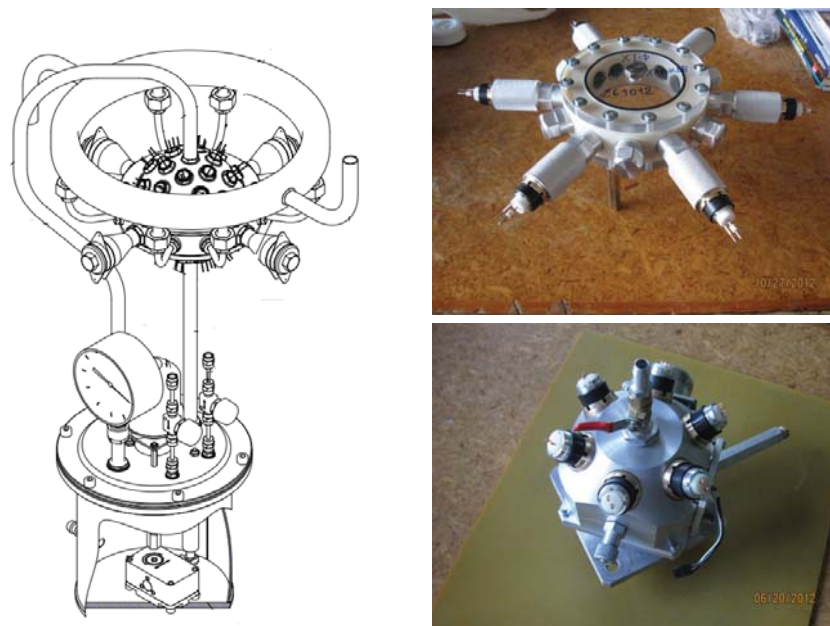


Fig. 38. Ultrasound cavitation installation and its “sonoreactors” with cylindrical and spherical symmetries.

objects on the mesoscopic scale, which consist, in turn, of indivisible structural units, namely monomers. It is possible to imagine nucleons as monomers and the nuclei of atoms and nuclear clusters as mesoscopic objects. The evolution of a system of monomers occurs through a reduction of the degrees of freedom and the formation of a scaling-invariant system of clusters with a definite level of coherence. On the fabricated experimental installations, we have realized the connection between the scales of molecular and nuclear levels with the help of entropic forces. As a result of the experiments with many fluids, we found the working fluids most promising for the synthesis. Among such media, we mention, in particular, partially correlated fluids with long molecules.

It is possible to obtain strongly correlated fluids with the help of a preliminary treatment of the working medium, by irradiating it, for example, with X-rays. At a similar treatment, ordinary water acquires a stable cluster structure, which is necessary for the enhancement of the order parameter on all scales and for the optimization of the processes of synthesis. It is clear that the same positive role in the synthesis will be played by the working media in the special states possessing the scaling and long-range correlations. In particular, we have measured, in a series of experiments, the sharp increase of the energy yield at the use of working media near phase transitions and critical points.

On installations of the type shown in Fig. 39 with ultrasound emitters at MHz frequencies, we found some promising working fluids, where the density of micron-sized bubbles in the working volume of a reaction chamber increases sharply up to $10^6 \div 10^8 \text{ l/cm}^3$ near phase transitions. In this case, the rate of registration of the number of photons emitted in the optical range increases by many orders from tens to 10^6 per second and more. The explosive increase

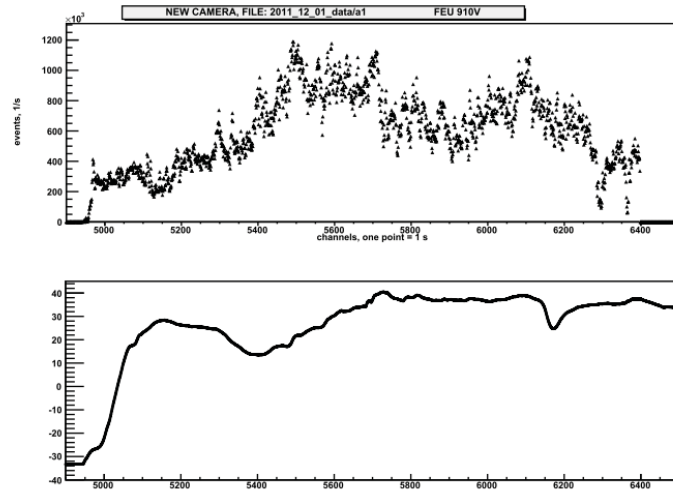


Fig.39. Generation frequency of sonoluminescent photons (upper curve) and variations of the temperature in glycerin (lower curve).

of the emission (Casimir light) testifies to great accelerations and the arisen connection of a substance with fluctuations of the physical vacuum.

The estimates of the ratio of the heat energy released in the experiment in these media and the energy consumed by the installation from a supply unit indicate the energy efficiency of the process with the coefficient of positive yield $Q \approx 6$ already at the present time.

In Fig. 39, we present the temporal changes of the temperature and the sonoluminescence intensity in glycerin. At the working volume of a reaction chamber of 300 ml, we registered a growth of the temperature by 50°C for at most 60 sec. This fact testifies to the existence of an additional energy source besides the cavitation energy introduced in glycerin. We refer the former with the above-described collective nuclear synthesis.

This result become possible due to the use of ultrasound drivers with the total power of about 300 W, preliminary treatment of working media, and realization of the process in a region of phase transitions. The experimental data allow us to conclude that the attained maximal value of the order parameter for shell structures lies in the interval $\eta \approx 0.008 \div 0.01$.

The theoretical analysis showed that the joint use of the acoustic drivers ensuring the generation of a sufficient number of nanobubbles in unit volume of the working volume of a reaction chamber and electromagnetic ones (electromagnetic pulses) in the process of synthesis in bubbles can initiate the fast evolution of the structure of the shells of bubbles to MQOs with the order parameter $\eta \geq 0.3$ and can lead to an increase of the energy efficiency of the process up to $Q \geq 25$.

6 Conclusion

The collective variables, ideas, and theories became used in nuclear researches almost at once after the appearance of nuclear physics in works by N. Bohr and J. Wheeler. But only in the last decades, the collective ideas were applied to the analysis of the structure of nuclear systems, which has led to the comprehension of a possibility to change their properties and parameters with the help of external actions possessing a low (by nuclear scales) energy. It became also clear that some general approach to a lot of physical effects, which are completely miscellaneous at first glance, should be developed.

In the cycle of works [1–3] and here, we made attempt to establish such general approach to the control of nuclear processes. On the basis of the analysis of the properties of nuclear clusters in the frame of the theory of nonlinear dynamical systems of strongly interacting nucleons and a change of the structure of the physical vacuum under the action of a coherent acceleration, we present new classes of action on the collective constraints and the nonlinear resonances in these systems.

We have proposed a new form of the description of a dynamics of nuclear systems with the help of a system of iterated functions. These new methods of description of nuclear systems and the action on them allowed us to develop the efficient scenarios of the initiation of collective self-organizing nuclear processes and the control of their energy directedness with the use of special actions (drivers) with low (by nuclear scales) energies, but involving the feedbacks in the interaction of a substance and the physical vacuum.

The dynamical processes and the correlations arisen as a result of the evolution of a self-organizing nuclear system define the possibility of a controlled formation of basically new stable nuclear structures and the conditions of their stability. The nuclear structures formed as a result of dynamical processes including a great number of particles (nuclei, nucleons, electrons) have exotic physical properties due to the existence of correlations in them, such correlations being induced by self-organizing processes in a substance and the physical vacuum.

It is worth noting that the evolution of the same open system can lead to different stationary modes, i.e., to different attractors. The presence of several attractors in the evolving system of a nuclear isomer allows one to control the evolution of this system, since the small perturbations of initial values on the boundary of one of the attractors cause the jump of a trajectory from one attractor onto another one and, by this, a change of the binding energy of of the resulting system.

Every system (e.g., a nonlinear medium) is characterized by a region of its parameters, where it is especially sensitive to external actions (to the control). Thus, we may say that any system has a resonance under its excitation, and the internal properties of media and their sensitivities are a key to the control of processes in them due to the fact that the resonance action on a nonlinear medium causes modes with sharpening in the controlled system.

Here, we have shown that the control of properties of a nuclear system by means of the targeted generation of electromagnetic and entropic fields leads to the synthesis of a spectrum of fractal isomers, which includes two most important components determining the observed activity of their mixture:

- Stable fractal isomers with a high specific binding energy exceeding the maximal values for nuclei without fractal structure. The physical and technological accessibility of their synthesis makes it possible to develop the promising safe sources of the nuclear energy on the basis of the initiation of the processes of fractal isomerization in stable nuclei even with medium values of mass numbers;
- Stable fractal isomers with relatively low binding energies and a small stability reserve. These isomers can be potentially used as accumulators of the nuclear energy in the processes of synthesis of fractal isomers and can become the efficient and environmentally safe nuclear fuel in the future.

We have presented the first experimental results realizing the methods of electromagnetic and entropic control of the systems with varying constraints in the processes of synthesis of isomers and the neutralization of the radioactivity of natural and technogenic isotopes. We have demonstrated the possibility to change the half-lives of isotopes in wide limits from several times to orders. The installations for the initiation of the processes of collective synthesis in condensed media with shell structures are designed and fabricated. On experimental laboratory installations for the cavitation synthesis with ultrasound excitation of bubbles in a fluid, we have attained the energy efficiency of the process with the coefficient of positive yield $Q \approx 6$.

Appendix. Algorithms of processing of experimental time series

The main components of the software that are essential for the determination of regions, where the time series are stationary, are the algorithms of regularization of time series and the algorithms of determination of an informational metric. These algorithms are presented in what follows.

Informational distances and definition of nonsteadiness of time series

In [22–23], the statistical structure of binary sequences with short (exponential) Markovian and long-range correlations was studied within the method of n -tuple Zipf analysis. The essence of this method consists in the determination of the normalized frequency $\omega(R)$ of appearances of a given “word” that is a binary combination of length n (n -tuple) depending on its rank R . The rank R is defined as the number of the word in the set of all possible words of length n (their number $N = 2^n$). This set is ordered by the diminution of the frequency $\omega(R)$. The rank $R = 1$ corresponds to the most frequent word, $R = 2$ corresponds to a word with a less frequency, etc.

It was found that, for the sequences with long-range correlation in a wide range of values of R (except for values close to the boundary values $R = 1$ and $R = 2^n$), the histogram of the frequency $\omega(R)$ decreases with increase in the rank R approximately by a power law

$$\omega(R) = A/(B + R)^\xi, \omega R^{-\xi}, \quad (1)$$

where ξ is called the Zipf index.

The index ξ can be estimated from the analysis of experimental data as the coefficient of slope of the Zipf histogram (plot $\omega = \omega(R)$ drawn on the double logarithmic scale).

A simple, approximately linear, dependence of the index characterizing long-range correlations and the Zipf index ξ was found in [23].

The importance of these distributions for our study consists in that they are very stable with respect to noises [24]. The parameters of the Zipf distribution are surely restored at the double excess of the noise amplitude over the signal level. In addition, the statistical distributions can be efficiently used as a basis in the determination of a pseudometric [23].

The distance between the time series $y_1(i)$ and $y_2(i)$ is determined in the present work with the help of the relation

$$d_s = \sum_i \left(h^{1/2}(\omega_{y_1}(i)) - h^{1/2}(\omega_{y_2}(i)) \right)^2, \quad (2)$$

where $\omega_y(i)$ is the Zipf–Mandelbrot distribution function of a time series, and $h(x) = -x^q \ln_q(x)$.

Here, we used Tsallis' generalization of the entropy (see, e.g., [5]), where a logarithm is replaced by its power generalization

$$\ln(x) \rightarrow \ln_q(x) = \frac{x^{1-q} - 1}{1 - q} \quad (3)$$

with some numerical parameter q . We note that $\ln_q(x)$ tends to the ordinary logarithm as $q \rightarrow 1$. The new formula for the q -entropy reads

$$S_q = - \sum_i p_i^q \ln_q(p_i) = \frac{1 - \sum_i p_i^q}{q - 1}. \quad (4)$$

A property of the q -entropy, which is significant for its use in diagnostics, consists in that the condition of maximum of the q -entropy leads to the Tsallis power distribution similar to the Zipf–Mandelbrot distribution, whereas the ordinary entropy (under condition of its maximum) leads to the equilibrium distribution function.

Regularizing properties of fractional derivatives

Let $x(t)$ be a real process running in the physical system under study. This physical process corresponds to another process $y(t)$ that is the measured time series. The latter differs from the former, since the measurements are performed with errors.

Let a dynamical system have the dimension of the embedding d_e and be described by an ordinary differential equation of power d_e . The operator causing the greatest instability in such equation is the higher derivative operator. Therefore, we will construct a regularizing operator in view of the regularizing operator for the higher derivative operator. For simplicity, let us consider the equation $D^{d_e}y = z$. The ill-posed problem of determination of the derivative of a quantity y by the time series is usually written in the form of the integral

equation $y = D^{-d_e}z$. We can show that the regularizing operator for the operator D^{-d_e} is a fractional integro-differential operator $R^\alpha = D^{d_e-\alpha}$ of the order $d_e - \alpha$ [21].

The regularizing operator plays the role of the inverse operator in the equation of a dynamical system. In other words, the term of the evolution equation with higher derivative should be replaced in our case by R^α (in order to obtain the stable unique solution as a result of the special smoothing of the initial series). However, the order of a differential equation (operator) can be preserved, if we introduce new smoothed variables with the use of the regularizing operator:

$$D^{d_e-\alpha}y = D^{d_e}z(t), \quad z(t) = R_f^\alpha y(t); R_f^\alpha = D^{-\alpha}. \quad (5)$$

The integral operator of smoothing of the variables R_f^α transforms the unstable solution of a dynamical system in a stable observable state. The regularizing parameter must satisfy, naturally, the following condition: if the dimension of an attractor is integer, rather than fractional, then the regularizing parameter must equal to zero. The choice $\alpha = \{d_f\}$ satisfies this condition. Here, $\{z\}$ is the fractional part of z , and df is any dimensional characteristic of the degree of randomness of a trajectory and the instability with respect to a perturbation of the initial conditions. It can be the Lyapunov dimension of a trajectory, correlation dimension, etc.

We will demonstrate the action of the algorithms of regularization on a model nonstationary time series (Fig. 1), which is generated with the use of the Weierstrass function W with the time-dependent correlation dimension

$$W(D_f(t), t) = \sum_n b^{n(D_f(t)-2)} \text{Cos}(b^n t). \quad (6)$$

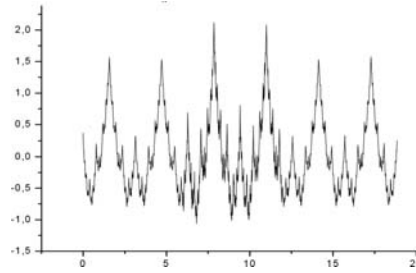


Fig. 1. Weierstrass function with the dimension varying in time from $Df = 1.33$ at the beginning of the series to $Df = 1.43$ in its middle and to $Df = 1.33$ at the end.

The series is obtained in the case where the dimension varies from $Df = 1.33$ at the beginning of the series to $Df = 1.43$ in its middle and to $Df = 1.33$ at the end of the time series. In Fig. 3, we show a trajectory in the phase space.

In Fig. 2 (a), we show a trajectory in the phase space, which is obtained with the help of the ordinary Tackens procedure. Figure 2 (b) demonstrates a phase trajectory as a result of the differentiation of a time series with the help of the preliminary smoothing and with the use of a fractional integral operator. It is seen that the procedure of regularization allows one to separate

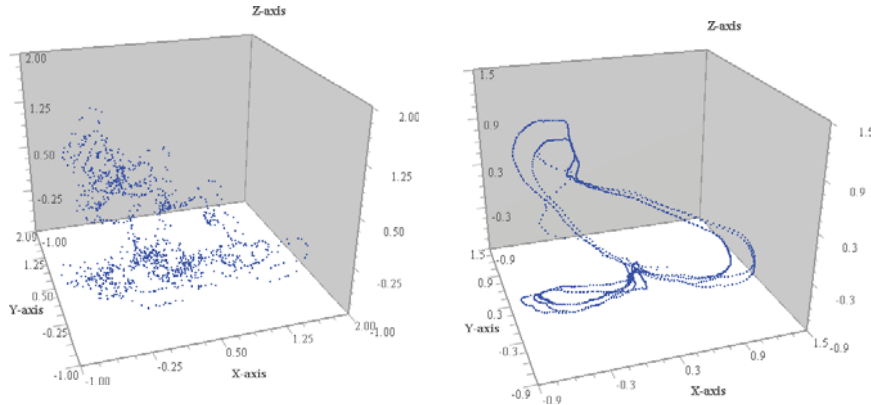


Fig. 2. Trajectory in the phase space for the time series in Fig. 2; (a) input trajectory; (b) regularized trajectory.

the quasistationary states. In order to clearly determine the boundaries of a steadiness domain, it is efficient to apply the pseudometric introduced in [23].

We see three domains with basically different behaviors of the distribution function. In Fig. 3, we give the dynamics of the informational distance of a time series to the testing sequence with the dimension $Df = 1.96$.

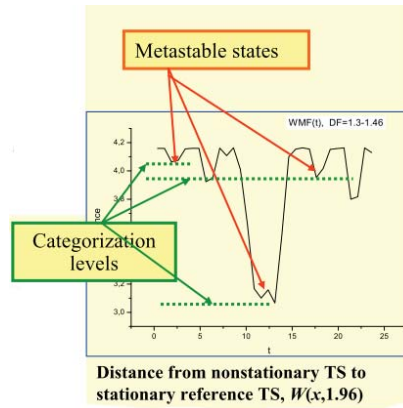


Fig. 3. Evolution of the informational distance between the tested time series and a supporting stationary series.

The presence of three different quasistationary states at the evolution of the system is clearly seen.

Classify the attractors

In order to classify the attractors, we use a statistical pseudometric [22–23], and the action of the procedure of regularization is well seen at the regularization of the Weierstrass function $W(t, D_f)$ with a fractal dimension D_f . In order to regularize this fractal function, we apply a regularizing operator to each term of the expansion of the Weierstrass function in a series. In this case, we take into account that $\alpha = D_f - 1$.

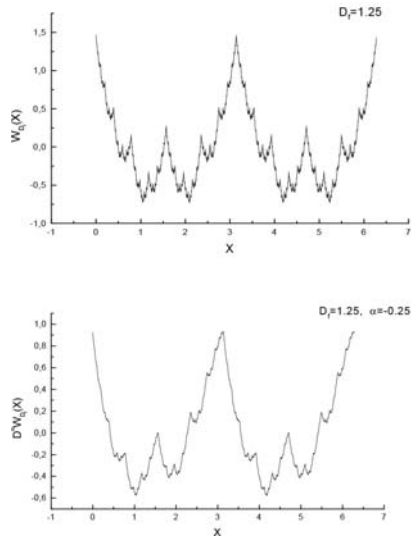


Fig. 4. Plots of the Weierstrass function with $D_f = 1.25$ and its regularization.

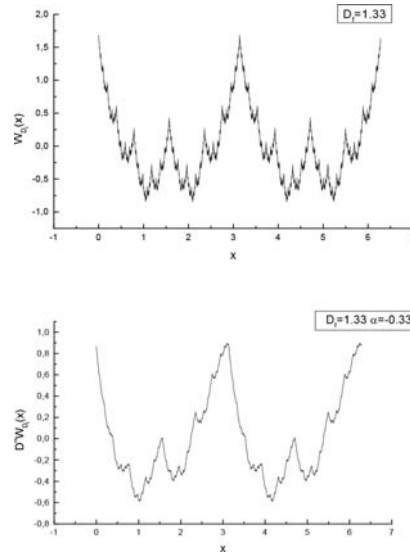


Fig. 5. Plots of the Weierstrass function with $D_f = 1.33$ and its regularization.

In Figs. 4 and 5, we show the Weierstrass functions and their regularizations for $D_f = 1.25$ and $D_f = 1.33$.

The smoothing of time series with the help of the regularization procedure leads to a change of the fractal dimension. However, the statistical properties of the studied series are slightly changed at the regularization.

We study the statistical properties of realizations with the help of the statistical pseudometric, which allows us to perform a classification of attractors. In Fig. 6, we present the distances of time series with various fractal dimensions and their regularizations to the time series that is the mantissa of the binary expansion of $\sqrt{5}$.

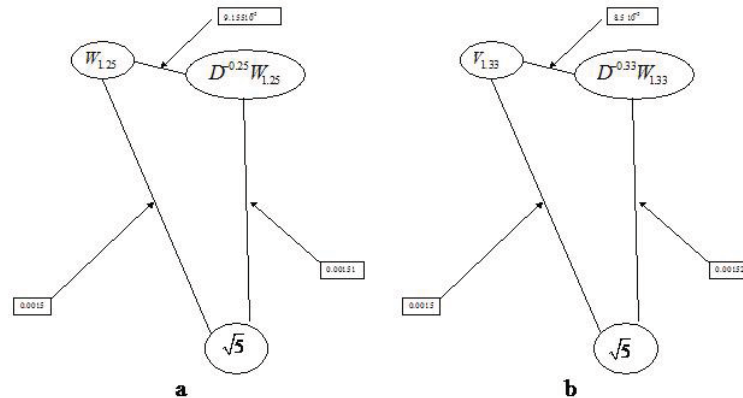


Fig. 6. Scheme of the distances between the Weierstrass function, its regularization, and the binary expansion of $\sqrt{5}$. (a) for $D_f = 1.25$, and (b) for $D_f = 1.33$.

The distances are normed to that between the binary expansions of the irrational number $\sqrt{5}$ and the transcendental number π . The distances between the time series and their regularizations are much less than the distances to the relevant expansion of $\sqrt{5}$. In other words, the regularization does not lead out the signals from their statistical class.

Studying the dependence of the “entropic distance” between the function and its regularization on the regularization parameter α , we can determine the optimal value of this parameter (regularizational dimension).

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