Tomashchuk O., Repeta V., Leshchynskyi O. Methods of lecturing the topic of "Solving exponential equations".

One of the main skills that students of high school must possess is a skill of solving exponential equations. Development of this skill during the teaching process can be done only by math teachers with a high enough level of corresponding theoretical and methodical preparation.

The article covers methodical aspects of lecturing the topic of "Solving exponential equations" in high school. Actively used students' books on "Mathematics" (standard level) and "Algebra and calculus" (profile level) subjects are analyzed for depiction of the stated topic. Main methods of solving exponential equations are systemized and methodical recommendations for math teachers on familiarizing students with these methods in the teaching process are given. In particular, the following methods of solving exponential equations are highlighted: 1) the method of reducing both hand-sides to powers with common bases; 2) the method of extracting common multiplier; 3) the method of reducing both hand-sides to powers with common exponents; 4) the method of substitution; 5) the method of using function monotonicity property; 6) graphical method; 7) the method of division by exponential expression; 8) the method of factorization.

Each method is described including its peculiarities and its application is illustrated with concrete problems. It is stated which methods of solving exponential equations are more suitable for classes of standard level and which should be considered in specialized classes. Examples of solving some nonstandard exponential equations and equations with parameters are provided. It is these exponential equations that should be considered in specialized classes. The ending of the article contains problems for home assignment. Doing these tasks will help them better encompass the main methods of solving exponential equations.

The material of this article will be useful for students on stages of preparation for the external independent examination on mathematics and for teachers who will teach students how to solve exponential equations.

Keywords: *mathematics, lecturing methods, equation, exponential equation, methods of solving exponential equations, nonstandard equations, equations with parameters.*

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MODERN THEMATIC PREPARATION FOR EIA IN MATHEMATICS IN UKRAINE: EQUATIONS AND INEQUALITIES

The relevance of the research on thematic preparation for the EIA in mathematics in modern Ukrainian realities is not in doubt. Based on the experience of systematization of the school course in mathematics, we have proposed dividing the whole course of mathematics into 10 thematic blocks: "Numbers and Expressions", "Functions", "Equations and Systems of Equations", "Inequalities and Systems of Inequalities", "Text Problems", "Elements of mathematical analysis", "Geometry on the Plane", "Geometry in the Space", "Coordinates and vectors", "Elements of probability theory and statistics".

In this paper, we propose thematic tests for two substantial blocks ("Equations" and "Inequalities"), as well as the answers to them. In addition, we solve the basic tasks of these tests and give methodological comments on these solutions. We are convinced that a properly organized thematic repetition of the school course in mathematics will allow teachers to excel in preparing pupils for independent assessment in mathematics.

Keywords: *IEA in mathematics, SFA in mathematics, thematic preparation, educational achievements of pupils, thematic tests, basic tasks, numbers and expressions, functions.*

Formulation of the problem. External Independent Assessment (EIA) of the quality of mathematics knowledge is now the main feature of assessing the quality of mathematical preparation for Ukrainian graduates. Also it is used for conducting the State Final Attestation (SFA) of academic achievements of senior school students, as well as a tool for competitive selection of applicants to Ukrainian universities. Thus, there is no doubt about the relevance and the need for research on various aspects of preparation to the EIA in mathematics.

One such aspect is the systematization and thematic repetition of the school mathematics course. Based on our many years of experience in training for EIA, during this repetition we divide the entire mathematics course into 10 thematic blocks: “Numbers and Expressions”, “Functions”, “Equations and Systems of Equations”, “Inequalities and Systems of Inequalities”, “Text Problems”, “Elements of mathematical analysis”, “Geometry on the Plane”, “Geometry in the Space”, “Coordinates and vectors”, “Elements of probability theory and statistics”.

It is this division that allows repeated repetition of the same material throughout the preparation process for the EIA. For example, the transformations of logarithmic expressions are repeated during the study of thematic blocks 1, 2, 3, 4, and 6. This permits the teacher constantly to keep the student in a tone, when he would forget something, but he can’t do this, because proposed thematic training system doesn’t allow it.

Analysis of current research. The problem of training students for EIA in mathematics is systematically considered in different scientific publications. Valentyna Bevz, Mykhailo Burda, Hryhoriy Bilyanin, Olga Bilyanina, Olga Vashulenko, Larysa Dvoretska, Oxana Yergina, Oleksandr Ister, Vadym Karpik, Arkadiy Merzlyak, Yevgen Nelin, Victor Repeta, Oleksiy Tomaschuk, Mykhailo Yakir and others constantly publish the results of their research in this area. During more then 15 last years, our author’s team has been constantly working to provide methodological support for the process of preparation to the EIA in mathematics. The theory and methodology of assessing the academic achievement of senior school students in Ukraine is described in the monograph [1]. For the repetition and systematization of the school mathematics course, we use the methodological set of manuals [2] and [3]. Previously, we have considered some aspects of thematic preparation for EIA, but since then the contingent of testing participants has changed significantly, as well as the methodological views of our author’s team on this problem are also evolved.

The purpose of the article. The purpose of this article is to provide methodological advice to teachers and tutors regarding the thematic training of graduates to EIA in mathematics. In particular, we present in this article two thematic tests related to the topics “Equations” and “Inequalities”, and also provide a solution of the some basic tasks of these tests with methodical comments for them.

Presenting main material. We suppose that in preparing for the EIA, it is advisable to refrain from a variety of task forms in the repetition and systematization of the material of each topic, limiting only to open-ended tasks with full explanation, as they are the most effective for teaching mathematics and feedback. However, after completing each of the 10 thematic blocks, it is natural to carry out a diagnostic thematic test in which to use all forms of test tasks inherent in the EIA math test.

Thematic test “Equations”.

Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.

1. Solve the equation $5x = 2$.

A	B	C	D	E
$\frac{2}{5}$	$\frac{5}{2}$	3	-3	10

2. Specify the root of the equation $x^2 - 10 = 0$.

A	B	C	D	E
100	5	10	$-\sqrt{10}$	-10

3. Specify the interval that includes the root of the equation $\sqrt{x+7,5}=3$.

A	B	C	D	E
$(-2;0)$	$(0;2)$	$(2;4)$	$(4;6)$	$(6;8)$

4. Solve the equation $\dfrac{x-8}{x+7}=0$.

A	B	C	D	E
-8	-7	0	7	8

5. Solve the equation $2+\cos x=2$.

A	B	C	D	E
$\varnothing$	$\dfrac{\pi}{2}+\pi n,\ n\in Z$	$\dfrac{\pi}{2}+2\pi n,\ n\in Z$	$\pi n,\ n\in Z$	$2\pi n,\ n\in Z$

6. Specify the root of the equation $2^x=20$.

A	B	C	D	E
$\log_2 20$	$\log_{20} 2$	10	0,1	18

7. How many solutions has the equation $\log_5 |x|=-x^2$?

A	B	C	D	E
None	One	Two	Three	More then three

In Task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

8. The graph of the equation $x^2+y^2=4$ is drawn on the figure. Using the graphic method, match the system of equations (1 – 3) to its number of solutions (A – E).

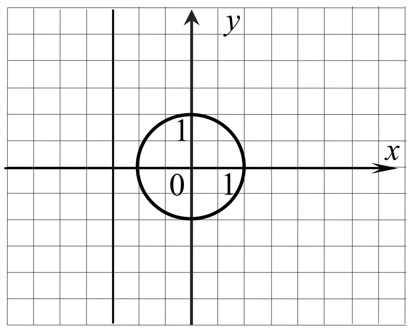


Figure to the task 8

System of equation

1 $\begin{cases} y=\log_{0,5} x, \\ x^2+y^2=4. \end{cases}$

Number of solutions

A None

B One

C Two

2 $\begin{cases} y = \sqrt{x}, \\ x^2 + y^2 = 4. \end{cases}$

D Three

E Four

3 $\begin{cases} y = \dfrac{1}{x}, \\ x^2 + y^2 = 4. \end{cases}$

Solve Tasks 9-11. Record the numeric answers you received *in decimal or integer*.

9. Find *the product* of all solutions of the equation:
   1) $x^2 + x - 56 = 0$; 2) $\left(x^2 + x - 56\right)\sqrt{3 - x} = 0$.

10. Solve the equation $\log_3(5x - 17) + \log_3 2 = \log_3(x - 5) + \log_3(x + 5)$. If this equation has only one root, write *it* in the answer. If the equation has several roots, write in the answer their *sum*.

11. Find the solution $(x_0; y_0)$ of the system of equation $\begin{cases} 2x - 3y = 7, \\ 4x + 5y = -8. \end{cases}$ In the answer write the value $x_0 + y_0$.

Solve Task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. Let the equation $\sin\left(\dfrac{x}{a}\right) = \dfrac{a}{4}$.

1) Solve this equation, when $a = 2$.
2) Solve the equation for any value of parameter a.

Answers to test "Equations"

1	2	3	4	5	6	7	8	9	10	11
A	D	B	E	B	A	C	1 – C; 2 – D; 3 – E	1) –56; 2) –24	9	–1,5

12. 1) $x = (-1)^n \dfrac{\pi}{3} + 2\pi n, n \in Z$; 2) For all $a \in (-\infty; -4) \cup \{0\} \cup (4; +\infty)$ the equation has no roots, for all $a \in [-4; 0) \cup (0; 4]$ the set of solutions is: $x = (-1)^n \cdot a \cdot \arcsin\left(\dfrac{a}{4}\right) + a \cdot \pi n, n \in Z$.

Solutions and comments to tasks of test "Equations".

Task 10 (term of the task see above). *Solution.* The set of valid values of the variable x is defined by following system: $\begin{cases} 5x - 17 > 0, \\ x - 5 > 0, \\ x + 5 > 0. \end{cases}$ On this set, by the properties of logarithms, we obtain the equation: $2(5x - 17) = (x - 5)(x + 5)$, it follows $x^2 - 10x + 9 = 0$. According to the Wiet theorem, the roots of this equation are $x_1 = 1,\ x_2 = 9$, but only root $x_2 = 9$ belongs to the set of valid values of variable. Thus, the correct answer is **9**.

Comment. This task focuses students' attention on the importance of writing down the conditions that determine the set of valid values of the variable *before* solving the logarithmic equation. It should be emphasized that it's not necessary to solve the system of inequalities (sometimes it can be more complicated than solving the equation), it's enough to substitute the equation roots to this system.

Task 12 (term of the task see above). *Solution.* 1) We write down the set of solutions of the equation $\sin\left(\frac{x}{2}\right)=\frac{1}{2}$ in form $\frac{x}{2}=(-1)^n\arcsin(\frac{1}{2})+\pi n, n\in Z$, it follows $x=(-1)^n\frac{\pi}{3}+2\pi n, n\in Z$. 2) It's obviously that for $a=0$ the expression on the left side of the equation is meaningless, and therefore the equation will not be resolved for such parameter value. Since the equation $\sin t = A$ has a solution only for $|A|\le 1$, then for all $a\in[-4;0)\cup(0;4]$ we obtain $\frac{x}{a}=(-1)^n\arcsin(\frac{a}{4})+\pi n, n\in Z$ or $x=(-1)^n\cdot a\cdot\arcsin\left(\frac{a}{4}\right)+a\cdot\pi n, n\in Z$. If $|A|>1$, that is, for all $a\in(-\infty;-4)\cup(4;+\infty)$, the equation $\sin t = A$ has no solutions, therefore the initial equation also will not be resolved. The complete answer to task 12 is written above in the answers to the test.

Comment. Obviously, item 1) of this task has a training character. It allows the student to remember the formulas for the solution of the simplest trigonometric equations and the methods for solving the trigonometric equations that are reduced to them. If the student correctly solved the equation from item 1), he (she) gets 1 point. Item 2) provides an analysis of the number of solutions of the equation depending on the values of the parameter. It is important to isolate the case $a=0$. If the student correctly deduced the number of roots of the equation in this case, then he (she) gets another 1 point. If the student correctly defined the set of values of the parameter a, in which the simplest trigonometric equation $\sin t = A$ has no solutions, then he (she) obtains another 1 point. Because the technical transformations of the expressions needed to get an answer in the case $a\in[-4;0)\cup(0;4]$ exactly the same as in item 1), then *only* if the student correctly wrote the *final* answer to the whole equation, he (she) gets another 1 point. Therefore, for correctly completed task 12, the student receives 4 points.

Thematic test "Inequalities".

Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.

1. Solve the inequality $4x<2$.

A	B	C	D	E
$(-\infty;2)$	$(2;+\infty)$	$(-\infty;0,5)$	$(0,5;+\infty)$	$(-\infty;-2)$

2. Solve the inequality $25>x^2$.

A	B	C	D	E
$(0;5)$	$(5;+\infty)$	$(-\infty;-5)\cup(5;+\infty)$	$(-5;5)$	$(-\infty;5)$

3. Specify the solution of the inequality $|2x|>16$.

A	B	C	D	E
-10	-8	0	5	8

4. Find the set of all solutions of the inequality $\frac{x+6}{x}<0$.

A	B	C	D	E
$(-\infty;-6)$	$(-\infty;0)\cup(6;+\infty)$	$(0;6)$	$(-\infty;-6)\cup(0;+\infty)$	$(-6;0)$

5. Solve the inequality $0,2^x \geq 0,2^3$.

A	B	C	D	E
$(0;3]$	$(-\infty;3)$	$(-\infty;3]$	$[3;+\infty)$	$(3;+\infty)$

6. Find the set of all solutions of the inequality $\log_3 x < -2$.

A	B	C	D	E
$\varnothing$	$\left(0;\frac{1}{9}\right)$	$\left(-\infty;\frac{1}{9}\right)$	$\left(0;\frac{1}{8}\right)$	$\left(\frac{1}{8};+\infty\right)$

7. The function $y = f(x)$ is defined and decreases on $(-\infty;+\infty)$ (see figure). Specify the set of all solutions of the inequality $f(x) \geq 0$.

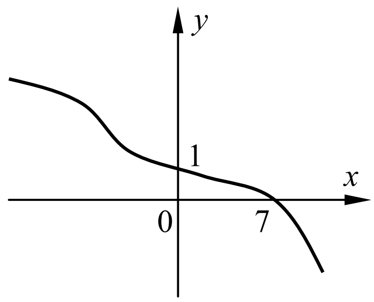


Figure to the task 7

A	B	C	D	E
$(-\infty;1]$	$(-\infty;7]$	$[0;+\infty)$	$[7;+\infty)$	$[1;+\infty)$

In Task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

8. The graph of the function $f(x) = 2 - |x|$ is drawn on the figure. Using the graphic method, match the system of equations (1 – 3) to its set of all solutions (A – E).

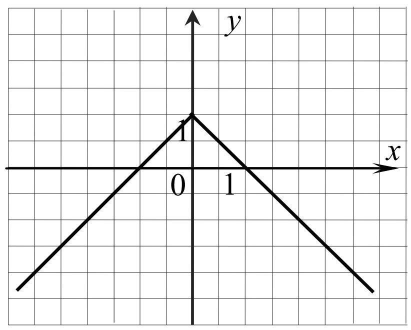


Figure to the task 8

	Inequality		*Set of all solutions*
1	$f(x) < x$	**A**	$(-\infty;-1)\cup(1;+\infty)$
2	$f(x) > x^3$	**B**	$(-1;+\infty)$
3	$f(x) > x^2$	**C**	$(-1;1)$
		D	$(-\infty;1)$
		E	$(1;+\infty)$

Solve Tasks 9-11. Record the numeric answers you received *in decimal or integer*.

9. Find the *sum* of all *integer* solutions of the inequality on the interval $[0;10]$: 1) $(18-2x)(x-6)\le 0$; 2) $\dfrac{(18-2x)(x-6)}{|x-8|}\ge 0$.

10. Find the *number* of *integer* solutions of the inequality $0 \le 5-x \le 2x+11$ on the interval $[-10;10]$.

11. Solve the inequality $4x^2+19x-5\le 0$. In the answer write the *number* of all *integer* solution of the inequality on the interval $(-10;10)$.

Solve Task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. Let the inequality $x^2-(a+2)\cdot x+2a\ge 0$.
    1) Solve this inequality, when $a=-3$.
    2) Find the set on all solutions of the inequality for any values of parameter a.

Answers to thematic test "Inequalities"

1	2	3	4	5	6	7	8	9	10	11
C	D	A	E	C	B	B	1 – E; 2 – D; 3 – C	1) 40; 2) 22	8	6

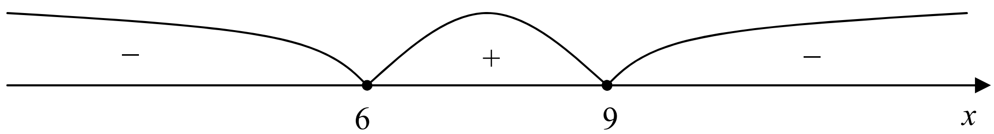
12. 1) $(-\infty;-3]\cup[2;+\infty)$. 2) For all $a<2$ the set of solutions is $(-\infty;a]\cup[2;+\infty)$, for $a=2$ the set of solutions is $(-\infty;+\infty)$, for all $a>2$ the set of solutions is $(-\infty;2]\cup[a;+\infty)$.

Solutions and comments to tasks of test "Inequalities".

Task 7 (term of the task see above). *Solution.* From the above figure we can see that the fragment of the graph, which is not below the abscissa axis, corresponds to all values $x\in(-\infty;7]$. Thus, the correct answer is **B**.

Comment. The ability by graphs of dependencies (for example, empirical ones) to find the value of an independent variable in which the value of a dependent variable satisfies certain conditions is extremely useful in the modern world. These graphs can be found in newspapers, magazines, textbooks, the Internet, etc. Task 7 prepares students for the ability to correctly analyze data presented in a graphical way, and thus helps them to acquire relevant competencies. Of cause, this is very useful in preparing for the EIA testing.

Task 9 (term of the task see above). *Solution.* 1) For $x_1 = 6$ and $x_2 = 9$ the left part of the inequality is equal to zero. We use the interval method by drawing the diagram of left-hand inequality side signs at the intervals formed for the variable x:



As we can see, the left side of the inequality is not positive for all $x \in (-\infty; 6] \cup [9; +\infty)$. The sum of all integer inequality solutions in the segment $[0;10]$: $0+1+2+3+4+5+6+9+10=\mathbf{40}$.

2) Since for all $x \neq 8$ the inequality $|x-8|>0$ is true, then the sign of the left side of the inequality does not change at each of the intervals shown in the figure above. For $x = 8$ the left part is meaningless and therefore the inequality solution will be the set $[6;8) \cup (8;9]$. The sum of all integer inequality solutions in the segment $[0;10]$: $6+7+9=\mathbf{22}$.

Comment. The interval method allows to solve inequalities if one can find the roots of the corresponding equation, and therefore it can be considered to be somewhat universal. This task permits to test, whether students can use this method in their practice. It is also important to emphasize to graduates that the short-answer task is considered to be solved *only when the final answer is received*. In this case, it is the sum of all integer solutions of inequalities at a given interval, not the set of solutions! And that sum should be found carefully without losing concentration. Students should also read carefully the part of the term of the task that is about the answer, that is, what exactly should be written to the answer. Such fragments of the term of the task should be distinguished in some way.

Conclusions. We suppose that a well-organized thematic training for the External Independent Assessment will allow teachers to keep their heartbeat on the problems encountered by students in the systematization and repetition of the school mathematics course. We hope that the suggested methodological advice will be of use to all specialists involved in this process. In future publications, we plan to continue detailed consideration of the repetition features for all of the above mentioned thematic blocks, as well as to regard for each such block a summary test with solutions to the basic tasks and provide methodical comments for them.

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Школьний О. В. Захарійченко Ю. О. Сучасна тематична підготовка до ЗНО з математики в Україні: рівняння і нерівності.

Актуальність досліджень, присвячених тематичній підготовці до ЗНО з математики, в сучасних українських реаліях сумнівів не викликає. Спираючись на багаторічний досвід систематизації та повторення шкільного курсу математики, нами запропоновано розбиття всього курсу математики на 10 тематичних змістових блоків: “Числа і вирази”, “Функції”, “Рівняння та системи рівнянь”, “Нерівності та системи нерівностей”, “Текстові задачі”, “Елементи математичного аналізу”, “Планіметрія”, “Стереометрія”, “Координати і вектори”, “Елементи комбінаторики і стохастики”.

У роботі ми наводимо тематичні тести до двох змістових блоків (“Рівняння” та “Нерівності”), а також відповіді до них. Крім того, ми розв’язуємо опорні задачі цих тестів та подаємо методичні коментарі до цих розв’язань. Ми переконані, що належним чином організована тематична систематизація і повторення шкільного курсу математики дозволить вчителям досягти успіху в підготовці учнів до незалежного тестування з математики.

Ключові слова. *ЗНО з математики, ДПА з математики, тематична підготовка, навчальні досягнення учнів, тематичні тести, опорні задачі, числа і вирази, функції.*

Школьный А. В., Захарийченко Ю. А. Современная тематическая подготовка к ВНО по математике в Украине: уравнения, неравенства.

Актуальность исследований, посвященных тематической подготовке к ВНО по математике, в современных украинских реалиях сомнений не вызывает. Опираясь на опыт систематизации и повторения школьного курса математики, нами предложено разбиение всего курса математики на 10 тематических смысловых блоков: “Числа и выражения”, “Функции”, “Уравнения и системы уравнений”, “Неравенства и системы неравенств”, “Текстовые задачи”, “Элементы математического анализа”, “Планиметрия”, “Стереометрия”, “Координаты и векторы”, “Элементы комбинаторики и стохастики”.

В работе мы приводим тематические тесты к двум содержательных блокам (“Уравнения” и “Неравенства”), а также ответы к ним. Кроме того, мы решаем опорные задачи этих тестов и даем методические комментарии к этим решениям. Мы убеждены, что должным образом организованная тематическая систематизация и повторение школьного курса математики позволит учителям преуспеть в подготовке учеников к независимому тестированию по математике.

Ключевые слова. *ВНО по математике, ГИА по математике, тематическая подготовка, учебные достижения учащихся, тематические тесты, опорные задачи, числа и выражения, функции.*