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Stochastic modeling of complex systems

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The aim of this short note is to give a brief description (oriented on non-mathematical audience) concerning some directions under an active development in the theory of complexity which are closely related to the scientific activity of the Interdisciplinary Research Center for Complex Systems (IRCCS) at the Dragomanov National Pedagogical University. This review is based on the talk given at the First annual meeting of IRCCS in September 2012. We will try to present certain key ideas in the area without an attempt to give an overview complete at any extend. It is why the references are restricted to minima just to give, for the motivated reader, some sources of a more detailed explanation. This note may be considered as an introduction into the more mathematical description of the corresponding problems of stochastic dynamics for complex systems which is presented in the next article of the present journal issue.

In all sciences, stochastic effects can rarely be ignored when we model, analyze or quantify any type of dynamics. In addition, the study of large complex systems (i.e., systems with large or infinite number of degrees of freedom) can often only be done using stochastic methods. The stochastic paradigm, recognizing the significant meaning of uncertainty, plays a fundamental role in dynamics researches of such systems. A fundamental concept is such that the randomness is just a reflection of a high complexity (due to A. N. Kolmogorov). In many cases, we use a probabilistic characterization as a reduced description of observed events. In the modern science, the stochastic modeling and statistical approach to the study of complex systems became a widely used technical tool. Note that the probabilistic interpretations of the real world notions are not always easily accepted. It is enough to mention a sceptic comment by Albert Einstein w.r.t. the probabilistic interpretation of quantum mechanics: “I am convinced that He (God) does not play dice”.

Stochastic methods play a key role in the study of complex systems. Representative examples of complexity in the real world are given by molecular motors, neuron networks, immune systems, social networks, financial markets, economics webs, communication networks, multi-particle systems in physics (gases, fluids, plasmas etc.).

Complex systems are those composed of many elements which mutual interaction gives rise to unexpected emergent phenomena. For example, the behavior of brain cannot be anticipated from the study of an isolated neuron. Similarly, superconductivity cannot be anticipated from the study of a single

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electron. Statistical mechanics, originally developed to study physical systems consisting of a large number of particles, provides tools and techniques that are well suited to study complex systems. The emergence of collective behavior is certainly not restricted to the physical sciences, but it is ubiquitous in nature, from biology, to social systems and economics. For this reason, the study of complex systems, being at the theoretical or empirical level, requires a truly interdisciplinary mindset. In particular, in applications to life sciences, the central problem is to develop an appropriate mathematical methods in the framework of statistical mechanics for complex systems. Such mechanics have to deal with heterogeneous ensembles of interacting agents and with the continual refreshment of that ensemble by novel and unpredictable types. A statistical approach to the stochastic dynamics of interacting particle systems makes the state evolution to be a main object of the study instead of the standard random path description for Markov processes [5]. The difference between both concepts became essential in the case of topologically complicated phase spaces as, e.g., configuration spaces in the continuum. In the most of known models of Markov dynamics in the continuum, we can not expect the existence of the process which starts from any initial configuration (i.e., microscopic state). On the other hand, such dynamics can exist for an initial distribution (i.e., macroscopic state) from the proper set of admissible states. This observation is an essence of the statistical approach.

Complex systems which appear in life sciences have several specific aspects. Description of these systems should take into account diversity and individuality of components, localized interactions among components, the outcomes of interactions used for replication or enhancement of components [6]. An essential typical property of biological systems (comparing with many situations in mathematical physics) is such that they are intrinsically out of the equilibrium.

In a more particular area of the spatial ecology, we have an active development of the concept of individual based models. In the mathematical terminology, each such model is a stochastic (Markov) process with events comprising birth, death and mobility for members of considered populations [3, 5].

Last years we observe an intensive and extensive development of a new area which may be called the physics of society. The complexity theory seeks to understand how the order and stability arise from the interactions of many agents. An appeared paradigm states that we can make predictions about a society even in the face of individual free will, and perhaps even illuminate the limits of that free will. As a typical aspect of the complexity theory, we stress that the physics of society is a science of humans' collective behavior. It is interesting to mention that the question about the roots of social events was already posed by Thomas Hobbes in his famous book "Leviaphan" (1651). Namely, he wrote: "We must ask not just how things happen in society, but also why". And here also the statistical point of view is a key concept. In fact, a shift from Newtonian determinism to statistical science makes a physics of society possible. Moreover, we shall accept that "the society itself is fundamentally a statistical phenomenon" [1].

The central technical problem is to develop an appropriate statistical mechanics that allows one to separate the knowable unknown from the truly unknowable. Such mechanics will have to deal with heterogeneous ensembles of interacting agents and with the continual refreshment of such ensembles by novel and unpredictable types. We would like to stress again that there is a principal difference between two possible points of view on the random evolution of complex systems. We put the macroscopic state evolution for the system as a fundamental notion in the statistical description of considered dynamics. In the terminology of probability theory, we are interesting in the Markov function dynamics instead of traditional study of Markov stochastic process starting from an individual points in the phase space (i.e., from microscopic states). Such random path description is more informative from the probabilistic point of view. But, as it was pointed out above, in the complex systems dynamics very often only statistical description is possible [5]. Actually, to understand clearly the appearance of statistical paradigm was a crucial step in the Boltzmann approach in the theory of classical gases.

The problem of relationships between various scales of description is one of the most important problems of the mathematical modeling of complex systems. The unity of knowledge (science) has been a widely discussed issue in the philosophy of science, as well as in specific scientific fields. A first step towards the unity of science is an establishment of relationships between different theories and models. Knowledge about the nature would be more reliable when relationships between different descriptions are more visible.

Perhaps, the most studied topics in physics are scaling phenomena. In this area we are based on the observation that the nature has a hierarchical structure with strongly separated levels. We believe in the following dogma: “Behavior at any level can be deduced entirely from the dynamics of the level below it, i.e. there are no new physical laws, only new phenomena, as one goes from atoms to fluids to galaxies” [2].

There exists a fundamental concept of three levels of the description for complex systems. In mathematical terms we are interesting in the links between the following levels:

– the micro-scale of stochastically interacting entities (cells, individuals, . . .) described in terms of linear Markov semigroups (or corresponding processes);

– the meso-scale of statistical entities realized in terms of nonlinear semigroups (evolutional families) related to the solutions of nonlinear nonlocal kinetic equations. Such structures have an interpretation as nonlinear Markov processes;

– the macro-scale of densities of interacting entities formulated in terms of dynamical systems related to local nonlinear equations (e.g., reaction–diffusion type equations).

One of the basic problems in each concrete model of interacting particle systems is to derive rigorously the description of transition from one level to another one [4].

All the history of the science demonstrates the presence and nontrivial interaction of two tendencies which we can call analysis and synthesis. They always exist in the science but their roles are different for particular periods of time. We clearly see that the tendency of a deep specialization (which was a dominant of the development during main part of 20th century) changed in last decades to an active motion in the direction of interdisciplinary researches. The theory of complex systems gives us a prominent example for this statement.

Mathematical methods play a crucial role in the successful realization of such motion. Note that a suspicious estimation of mathematical style of thinking was quite extended in the society. We remember that J. W. Goethe said: "Mathematicians are like Frenchmen: whatever you say to them they translate into their own language, and forthwith it is something entirely different". And it was not only an opinion of the great poet. Such giant of natural sciences as V. I. Vernadsky wrote in his monograph "La Biosphere" (1926) the following: "Considered in the abstract time and space of mathematics, Life is a fiction, a creation of our mind which is very different from reality". But later the quantum revolution in physics, an implementation of mathematical methods in biology, ecology and other life sciences, developments in informatic technologies and many other great scientific evens made the leading role of mathematics in the interdisciplinary researches obvious.

Here I would like to give a reference to the talk of I. M. Gelfand at the conference "The Unity of Mathematics", Cambridge MA, USA, 2003. He was not only the great mathematician with highest level results in the pure mathematics, but also the Director of the Laboratory of Mathematical Biology at Moscow State University. In particular, I. M. Gelfand said: "An important side of mathematics is that it is an adequate language for different areas: physics, engineering, biology. Here, the most important word is adequate language. I can give you examples of adequate and non-adequate languages. For example, to use quantum mechanics in biology is not an adequate language, but to use mathematics in studying gene sequences is an adequate language... My search for an adequate language is based on my work in applied mathematics. Mathematical language helps to organize a lot of things".

One of the aims in general theory of complex systems is an elaboration of mathematical models for observed notions appearing in several particular areas of the science. There is a popular name "real world models" for the characterization of this direction. Interacting particle systems are used often as a technical tool for the construction and the study of these models. They are used as basic ingredients in developing concrete models in physics (gases, fluids, condensed matter), chemical kinetics, biology, ecology (individual based models), medicine (tumor growth models), sociology, economics (agent based models) etc.

Here we arrive in a very delicate area which may be called the art of modeling. It is clear, that a mathematical model which takes into account too much details of the considered notion may be, as a result, very far from a rigorous mathematical study. On the other side, mathematically convenient simplifications may lead to an empty set of possible applications. The necessary balance here was formulated by Albert Einstein: "Everything should be made as simple as possible, but not simpler". Another question is the following: what we may expect from a particular model? In fact, we must remember that, due to Mark Kac, "models are, for the most part, caricatures of reality, but if they are good, then, like good caricatures, they portray, though perhaps in distorted manner, some of the features of the real world". And a British mathematician George Box added: "All models are wrong; some models are useful". In any case, models describe observed (or expected) notions rather than real systems. There is an additional notable moment. A "good model" may became a part of scientific reality itself with its own life and development. A prominent example here is the celebrated Ising model. An exiting attempt of the complexity science is to recognize such particular gold fishes in the flow of considered models and to analyze them with all respect and the careful attention they are deserved.

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STATISTICAL APPROACH FOR STOCHASTIC EVOLUTIONS OF COMPLEX SYSTEMS IN THE CONTINUUM

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Abstract. We present a general background for the study of complex systems in the continuum and explain the mathematical tools to deal with stochastic evolutions in the continuum. The statistical description of Markov dynamics of complex systems in the continuum is described in details. The review of recent developments for birth-and-death evolutions is given.

1 Complex systems in the continuum

In recent decades, different brunches of natural and life sciences have been addressing to a unifying point of view on a number of phenomena occurring in systems composed of interacting subunits. This leads to formation of a interdisciplinary science which is referred to as the theory of complex systems. It provides reciprocation of concepts and tools involving wide spectrum of applications as well as various mathematical theories such that statistical mechanics, probability, nonlinear dynamics, chaos theory, numerical simulation and many others.

Nowadays complex systems theory is a quickly growing interdisciplinary area with a very broad spectrum of motivations and applications. For instance, having in mind biological applications, S. Levin [40] characterized complex adaptive systems by such properties as diversity and individuality of components, localized interactions among components, and the outcomes of interactions used for replication or enhancement of components. We will use a more general informal description of a complex system as a specific collection of interacting elements which has so-called collective behavior that means appearance of system properties which are not peculiar to inner nature of each element itself. The significant physical example of such properties is thermodynamical effects which were a background for creation by L. Boltzmann of statistical physics as a mathematical language for studying complex systems of molecules.

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